HISTORY OF EVOLUTION OF CONCEPT OF FUNCTION

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ABSTRACT

Numerous branches in mathematics deal explicitly or implicitly with numbers: mathematical research takes into account numbers of one, two or n variables, examining their properties as well as those of their derivatives; differential and integral equation theories are directed at solving equations under which the undefined exists; functional analysis deals for functional spaces; and numerical analysis operates. Many areas of mathematics deal with ideas that represent generalizations or outgrowths of the concept of function; algebra, for example, recognizes procedures and relationships, and mathematical logic studies of recursive functions.

Within this essay, we will expand on these points and seek to give the reader a sense of the enthusiasm and difficulty faced by some of the greatest mathematicians of all time within trying to get to grips with the fundamental definition of function that we now embrace as commonplace.

INTRODUCTION

The development of the idea of function can be seen as a tug of war between two components, two mental images: the geometric (expressed in the form of a curve) and the algebraic (expressed as a formula — first finite, and then having indefinitely several terms, the so-called "analytic expression"). Thus, a third dimension, namely the "logical" concept of function as correspondence (with a mental picture of an input-output machine) is entered. The algebraic conception of function is progressively abandoned in the wake of this development. A new tug of war soon develops between this novel "logical" ("abstract," "synthetic," "postulation") conception of function and the old "algebraic" ("concrete," "analytic," "constructive") conception (and is, in one form or another, still with us today).

It has long been claimed that functions would constitute a central principle in mathematics of secondary school and the current curriculum orientations specifically stress the significance of functions (National Council of Mathematics Teachers, 1989). Based on the prevailing mathematical point of view, the notion of function may be interpreted in a variety of different forms, each with specific educational implications.

This article explores some of the most influential facets of idea of function history,1 investigates its connection to other disciplines, and addresses its application in the analysis of real-world scenarios. Ultimately, the question of a didactic method is addressed, with specific attention provided to the essence of the working principle underlying the students' activities and the function of various modes of representation.

Keywords: Concept of function, history, evolution of function in mathematics.

THE HISTORICAL DEVELOPMENT OF THE FUNCTION CONCEPT

For this analysis the central mathematical notion is the definition of feature. Therefore, it seems fitting to present, from a mathematical viewpoint, a quick overview of the historical evolution of this definition. The definition of function has been considered one of the basic principles of mathematics, at least since the early 20th century (Sierpinska, 1992). Therefore, it may be quite shocking that it has been consistently researched for just around 300 years. As Kleiner (1989) puts it: "The history of the idea of function dates back 4000 years; 3700 of these consists of anticipations." Indeed, it was stated that "the definition of function is one of the defining features of 'new' as opposed to 'classical' mathematics". Sierpinska (1992) concludes that "the notion of structure should be seen as a consequence of the human effort to come to grips with the improvements perceived and encountered in the real universe." Of course, also in ancient times, practical concepts of co-variation and connections between magnitudes were used, for example, to construct astronomical charts and tables. This was not deemed part of mathematics though (Sierpinska, 1992). Ancient mathematics also ignored the algebraic prerequisites required for the creation of a definition of action (Kleiner, 1989). That was finally to change. During the two hundred years leading up to the research of Newton and Leibniz in the 1680s, there were hundreds of scientific advances paving the way for calculus and the origins of the definition of action as it is commonly known today. Of example, the research of the likes of Kepler and Galileo in astronomy contributed to the development of the study of motion as a key science issue (Kleiner, 1989). This time also saw the emergence of symbolic algebra, and the creation of a solid connection between algebra and geometry, through the research of Descartes and others. The Newton and Leibniz calculus, though, was not only a calculus of numbers, instead rather quantities connected to physical structures, and the framework in which they conducted their restricting processes was spatial, not arithmetic (Guicciardini, 2003). As Leibniz coined the word 'shape' it was "to classify a curve-related geometric entity" (Kleiner, 1989). Not after Euler 's dissertation in the mid-1700s did calculus continue to be used as an analysis of numbers, not curves.

In 1718 Johann Bernoulli gave the first description of the idea of function: "One calls here Function of a vector a quantity consisting in some manner whatsoever of this component and constants" (Rüthing, 1984). Of course, this description was rather imprecise and Euler changed much. Common to these early definitions was that they required an assigning to give the function. A vigorous debate on the issue of defining the motion of a vibrating string based on the definition of a function as, on the one side, the answer to a physical problem, and, on the other side, an empirical term (for a more thorough explanation, see Kleiner, 1989) prompted Euler to modify his understanding of the meaning of action. He gave the following description of the function principle in 1755: "a quantity can only be considered a function if it relies on another quantity in such a way that, if the latter is modified, the former undergoes change itself" (Sfard, 1992). Rather than depending on empirical terms, this concept incorporates the principle of quantity covariation (Thompson, 1994). The description of Euler using Sfard's (1991) process-object duality language is explicitly process-oriented.

The mathematics quickly evolved in the 18th and early 19th centuries but the core principles were treated in a less than thorough way. Once Fourier, influenced by the question of heat conduction in a thread, presented his dissertation on trigonometric sequence in 1822, it questioned the existing theories regarding the principles of calculus. In passing it can be stated that Malik (1980) allows the statement that the question of heat conduction is mathematically close to the question of the vibrating string, except that the simple geometrical understanding of the string issue rendered it impossible, and perhaps pointless, for mathematicians of the 18th century to make the logical leap taken by Fourier and others a hundred years later. Yet it became obvious that Fourier 's arguments lost logical precision. It was often observed by university mathematicians teaching science to be a daunting activity without a sound intellectual basis (Lützen, 2003). A thorough re-treatment of the principles of calculus was necessary, and such mathematicians as Cauchy, Dirichlet and Weierstrass pursued this undertaking in the first half of the 19th century. With regard to the principle of feature, we have seen that Euler tried to escape the near logical relation between functions and analytical expressions already in his 1755 interpretation. This was the case both for the Cauchy and Fourier concepts. Nevertheless, both often focused in reality on the concept of feature as an abstract term, particularly in the way they talked of functions, or in the way they interpreted facts (Kleiner, 1989, Lützen, 2003). In truth, "it was very popular for mathematicians of the early 19th century to describe functions in a general manner and then indirectly or expressly assign different additional properties to them in the context of the arguments" (Lützen, 2003). The first mathematician to regularly refrain from this was Dirichlet (Lützen, 2003), and it was he who offered what may be considered as the first modern interpretation of the feature concept: "If a variable y is so connected to a variable x that, if a numerical value is allocated to x, there is a law that specifies a specific value of y, then y is said to be a feature of the ind In a way this concept may be considered as less universal than the one cited by Euler above, speaking of "law" rather than "dependence," although at the same time the notion of co-variation is less apparent, rendering it a reference to current meanings. This is rendered much clearer by the fact that description of one-value is the first concept. According to Even (1990), one-value and arbitrariness are the two basic characteristics in the current context of the definition of purpose.

THE EVOLUTION OF THE NOTION OF FUNCTION

The definition of function in all mathematics is properly considered one of the most important. Quantities tend to occur. However, the advent of functions as a distinctly individualized definition and as a topic of analysis in its own right in mathematics science is very recent, dating back to the end of the 17th century.

The appearance of a notion of function as a personalized mathematical object can reveal extremely small calculus beginnings. Descartes (1596-1650) made it plain that an equation in two variables, described geometrically by a curve, implies a relation between variable quantities. The derivative concept came about as a way to locate the tangent to some point in this curve.

Newton (1642-1727) was one of the first mathematicians to demonstrate how functions in infinite power series could be shielded, thereby enabling infinite processes to interfere. He used "fluent" to denote independent variables, "relate quantities" to signify multiple variables, and "genita" to apply to amounts derived from those using the four operations.

Therefore, the notion of function was associated with the notion of analytical expression in action. Soon this formulation was interpreted to lead to many incoherence's; in reality, multiple separate analytical terms may describe the same feature. The approach also yielded significant limits on the types of functions that could be taken into account. Throughout today's jargon, we can conclude that the definition of Euler contained only the analytic functions, a restricted subset of the very tiny class of continuous functions. Conscious of these limitations, Euler suggested an alternate description that at the time did not gain much interest. With regard to conventional mathematics, the identity of functions with analytical expressions would remain unchanged in the 18th century. However, throughout the 19th century, the notion of function experienced successive expansions and clarifications which profoundly changed its essence and significance. Among them is Oresme (1323-1382), who established a geometric theory of latitudes of types. Any basic concepts of independent and dependent variable, in his theory. A big drive for the expansion of the definition of feature came first from the

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

Where y, the dependent variable, represents the displacement from the direction of the equilibrium, x is the distance from the origin and t is the time. The debate, influenced by d'Alembert (1717-1783), concerned the functions that would, in essence, be seen as solutions to this problem, with mathematicians like Euler and Daniel Bernoulli (1700-1782) arguing for more general solutions. Another significant contribution to the practical evolution comes from Fourier 's works (1768-1830), which were dealing with the question of heat transfer in material bodies. Temperature was used by Fourier as a function of two quantities, namely time and space. At some point, he conjectured that a creation of some function in a trigonometric sequence may be obtained at a reasonable interval. Yet Fourier never provided any statistical evidence of his argument. The question was later picked up by Dirichlet (1805-1859), who proposed the necessary criteria so that a Fourier series may reflect a function. To do so, Dirichlet wanted the definition of feature to be isolated from its theoretical image. He did so in 1837, setting the function description in terms of an arbitrary relationship between numerical sets representing variables. Then a function is a relationship of two variables such that one and only one value of the dependent variable is related to the value of the independent variable. Only Dirichlet provided this well-known illustration of a feature which is discontinuous in all domain points [0,1].

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Another significant contribution to the development of nature comes from Fourier 's works (1768-1830), which were dealing with the heat distribution issue in material bodies. Temperature regarded in Fourier as a function of two factors, namely time and space. At some level, he conjectured that every feature in a trigonometric series may be formed within a suitable interval. But Fourier never provided his claim to a statistical proof of. Dirichlet (1805-1859) later took up the problem, formulating the sufficient conditions for a function to be represented by a Fourier series. Of this, Dirichlet had to distinguish the feature definition from its conceptual representation. In 1837 he did this, setting the function description in terms of an arbitrary relationship between variables describing numerical sets. A feature then becomes a relationship of two variables such that there is associate done of every value of the independent variable and just one value of the dependent variable. Only Dirichlet provided this well-known illustration of a feature which is discontinuous in all domain points [0,1].

0 Where x is a rational number

1otherwise.

The notion of function began to advance with the growth of the set theory, pioneered by Cantor (1845-1918). Function was extended in the 20th century to include all arbitrary correspondences satisfying the uniqueness condition between sets, numerical or non-numeric.

Type has started to grow. Mathematicians shifted from the notion of connection to the notion of a link. A strong resemblance to the notion of feature constitutes in category theory a basic definition. For example, in computation theory, as in l-calculus, a function is regarded not as a partnership but as a law of computation.

The notion of function was used in its infancy to define interaction between geometrical entities. Form gained a central position in the history of mathematical analysis through its connection with the research of logical expressions. Commented Youschkevich (1976/77) on the historical position of this relationship, It was the analytical method of implementing functions that revolutionized mathematics and, owing to its exceptional effectiveness, established a central position for the notion of feature in all This mixture of analytical expressions and geometrical structures in turn proved itself to be so extremely productive that it still informs current mathematical practice.

DIFFICULTIES IN THE CONCEPT OF FUNCTION

As the human brain absorbs new material, it attempts to categorize it with its current knowledge base and synthesize it. Nevertheless, in these actions of learning, students continue to create incorrect conclusions and expectations about new theories, particularly as regards complicated concepts such as functions. Many misunderstandings confuse the interpretations of the students' roles.

Students also assume that to be properly called a function, functions must be constant and differentiable. Anna Sierpinska describes this phenomenon, "As usually the first instances of functions found by a student are continuous everywhere, non-differentiable in the most limited number of points, consisting of one piece of a curve in the graphic representation provided by a single formula; these unusual functions constitute, in the mind of the student, a prototype of a function"[21]. Students may pause before acknowledging it as a function when faced with a function beyond the examples they have been taught. Most students focus their interpretation of what a feature is, rather than the concepts they were given, on their reserve of instances. Professor Anna S fard examined 22-25-year-old university students who had completed a simple mathematics course covering introductory set-theory, geometry, and calculus for her thesis on the concepts of functions of pupils. She finds that many students are unfamiliar with piecewise functions and prefer to interpret case-defined expressions over separate domain parts as multiple functions instead of one [19]. She asked the students in a questionnaire to state that the following definition represents a function (x and y are real numbers):

If x is an even number then $y = 2x + 5$
Otherwise (x is an odd number) $y = 1 - 3x$.

About 50% of the test community agreed that this statement represents a feature while others readily proposed it represent two different functions. It's often challenging for certain students to agree that a graphical depiction of a discontinuous curve reflects one function rather than many. Such a perplexity is normal provided that even great mathematicians like d'Alembert did not recognize the functions of split-domain. In addition, the notion of a non-differentiable function was so fresh and contested that mathematician Hermite proclaimed in 1893, "I turn away from this lamentable evil of functions which have no derivatives with terror and horror".

Similarly, certain mathematicians claimed that there was a law or formula behind functions, which contributed to a second widespread fallacy. Sierpinska relates that "algebraic ability accompanied by a belief in the ability of algebra to solve almost automatically many types of problems can be an obstacle to understanding the general concept of function". Students accustomed to dealing with algebraic expression notation functions that have difficulties understanding that functions may be randomly built. Much as mathematicians before Dirichlet were not seriously embracing the idea of a function as an arbitrary relationship, because of its abstract existence, students struggle to grasp the notion. Such a concept was not conjured out of desperation or immense utility to the outer world of mathematics, but its creation became vitally essential to those who sought to recognize and gain a deeper knowledge of functions. In 1899 Poincar'e shared his dissatisfaction with the latest advances of functional philosophy, "In ancient times when a latest concept was developed, it was for a practical purpose; nowadays one deliberately invents them to expose flaws of our fathers' logic, and we can deduct from them that only". Although abstract functions can appear irrational, their presence reinforces the underlying idea that algebraic expressions are merely a way of representing functions. Nevertheless, students prefer to see formulas as objects per se rather than as manifestations of certain entities. Sfard had posed the following truthful or false questions to students in her study:

1) Every function reflects a certain regularity (it is difficult to fit the x and y values in a totally random way).
2) A certain computational formula will articulate any function

Only 6 percent of students replied that both claims were incorrect, indicating that most students assume there must be a function-specific algorithm to be true. Sfard indicates that "the students not only tend to worry of method functions rather than permanent artifacts, but they also assume that the systems must be algorithmic and relatively simple". The preference for algorithmic consistency also illustrates the hesitancy of the students to embrace discontinuous and non-differentiable functions. Sierpinska suggests that introducing students to one function represented by two different formulae may help students distinguish between the function itself and the "analytical tools" used to describe it. Unfortunately, it's challenging for students to agree that algorithms that look different but that generate the same values are essentially the same function. If faced with the various functions of algorithm:

$\mathbb{N}$ to $\mathbb{N}$: $f(x) = x^2$
and the recursively defined
$g(0) = 0,\ g(x+1) = g(x) + 2x + 1$

Students refused to accept they were equal even though they generated the same values. This confusion is related to the unfamiliarity of the students with the organized functional pair description. We cannot realize that two distinct algorithms generating the same set of ordered pairs are essentially the same feature, so they cannot distinguish their interpretation of functions as laws.

Once the students are first exposed to the definition of feature, the vertical line method is also learned to determine if a graph is a feature. The exam instructs students to push the graph

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they are checking over a vertical axis. Unless the line only crosses two graph points at one time, then the graph is not a function. This approach is a beneficial guide for young students who have difficulty knowing what it requires to provide individuality for the feature but it does not deter students from mixing domain and context meanings particularly though they are older. In addition, several students overcompensate and retain the illusion that a one-to-one communication must be in functions. Ed Dubinsky and Guershon Harel explain, "It is exceedingly normal for topics at all stages to have trouble with this peculiar state, and to equate it with the notion of one-to-one".

This tendency may be due to the lack of capacity of the students to imagine functions. Although they can remember and appreciate the vertical line method, without a graph on which to use it, the method is of little consequence. Eisenberg is saying this. Students have a clear propensity to think about algebraic rather than visual functions "while visualization can be extremely helpful. Some claim that students avoid visual representations because visual processing involves skills at a higher degree than analytical processing. Although logical analysis sometimes entails just one degree of abstraction from a concept to quantitative figures, visualization includes the capacity to analyze a term, identify patterns and move all the information to a graphic medium. Interestingly, in his analysis of teachers' awareness of functions, Alexander Norman found that teachers appear to focus on and favor graphical representations of functions, in particular when deciding the functionality of a phrase as opposed to the students. He says teachers have also provided a large amount of access to multiple forms of tasks. For regular graphs 15, they are confident and know how to address a number of questions from such graphs. Visualization can be intimidating for students who have had considerably less exposure to standard graphs, and can feel quite foreign. Only enhanced graph and visual access will help the students resolve this resistance.

Such assumptions are clearly a result of the inadequate perception of the meaning of action by the students. Although confusion is inevitable when students study new ideas, educators' goal is to help students reach the maximum degree of comprehension in the shortest period of time available. Careful analysis of the cognitive processes and capacities of the students, as well as exposure to external influences that lead to confusion, are important for achieving this objective.

THE CONSIDERATION OF NEW DEVELOPMENTS OF FUNCTION IN MATHEMATICS

Nowadays, as in the past, mathematics is no longer so closely related to the physical sciences. It has seen a substantial rise in its areas of research, being an tool for the analysis of biological science, human and social sciences, industry, media, engineering and technology phenomena and circumstances. Mathematics is an important means of classification, interpretation, estimation, and supervision.

Both mathematical methods presuppose the notion of pattern. A mathematical model is a depiction by means of relationships and arrangements designed to depict the basic elements in a given circumstance while intentionally omitting secondary elements. A mathematical model may assume many types, but it is typically made up of variables, interactions between these variables, and their respective change levels.

The method of constructing a mathematical model includes a variety of steps, from the scenario to the explanation of mathematics and back to the scenario. It may take several cycles to yield a satisfactory result. Various actions form an important part of this process:

- Definition of goal cognitive elements;
- Collection of items, associations, etc. related to certain purposes;
- Idealizing certain concepts and relationships in a manner suitable for mathematical representation;

Complex processes are among the concepts most commonly modelled in mathematics. The basic variables in these models indicate the state of the system, representing quantities known as "accumulations" that may increase or decrease, such as to origin distance, mass, kinetic energy, liquid volume, number of living organisms, and so on. Equations involving the rate of change of these quantities indicate their variations over time.

Yet versions may be of various kinds. In models of spatial representation, how various or quantities of items are spread and how they travel through space. Discrete stochastic models describe series of events that, according to certain probability functions, take place in time.

For both of these situations, observing the results of the different element that affect the condition is also of considerable importance. To do this effectively, we need to establish functional relationships involving the parameters representing those factors and the model 's fundamental state variables.

Whatever their existence, the notion of numerical structure as a correlation between variables is of considerable significance in the design and analysis of mathematical models. Consequently, it remains a central notion in mathematics applications today.

Precalculus Developments It was not until the beginning of the 18th century that the notion of function in explicit form emerged, though implicit manifestations of the concept date back to about 2000 BC. The main reasons the concept of function didn't come up earlier were:

- Lack of algebraic preconditions — tackling the continuum of real numbers and developing symbolic notation;

- No motivation. Why define an abstract notion of function unless one had a lot of examples to abstract from? Over the course of about two hundred years (ca. 1450–1650), a number of developments have occurred which were fundamental to the rise of the concept of function:
- Expansion of the number principle to cover actual and (to a degree) also complex numbers (Bombelli, Stifel, et al.);
- A symbolic algebra (Viète, Descartes, et al.) is created;
- Observing motion as a core scientific question (Kepler, Galileo, et al.);
- Algebra and Geometry marriage (Fermat, Descartes, et al.).

The 17th century witnessed the rise of modern mathematicised research as well as the discovery of analytical geometry. Both of these developments suggested a continuous, dynamic view of the functional relationship as opposed to the static, discrete view held by the ancients.

The key elements in the mixing of algebra and geometry were the introduction of variables, and the expression of the relationship between variables through equations. The above presented a vast range of explanations for researching curves (potential functions) and set the final stage for presenting the definition of structure. Everything was absent in an equation was the description of the independent and dependent variables:

Variables don't work. The definition of feature suggests a unidirectional interaction between the variable "neutral" and the variable "dependent." Yet in the case of variables which occur in mathematical or physical problems, such a separation of roles may not need to occur. And as long as one of the variables involved is not given a special independent role, the variables are not functions but merely variables.

The calculus Newton and Leibniz established did not have the shape students see today. In fact, it was not a functional calculus. The central subjects of research in mathematics of 17th century is curves (geometric). (For instance, the cycloid was developed geometrically and researched thoroughly long before it was offered as an equation.) In addition, the study of the 17th century emerged as a series of methods for solving curve problems, such as identifying curve tangents, curve regions, curve lengths and point velocities. Because the questions that gave rise to the calculus were in essence geometric and kinematic, and because Newton and Leibniz were concerned with manipulating the wonderful device they had developed, it would require time and thought before the calculus could be recast in algebraic form.

Newton's "fluxion process" refers to "Fluent’s" and not to structures. Newton terms the variables "Fluent’s"—the concept (as in Leibniz) is geometric, of a" flowing "point around a curve. The main contribution Newton brought to the advancement of the definition of action was the usage of power series. These were essential to the concept 's subsequent growth.

Introduction of Euler in Infinitorum Analyzing. We are seeing a steady isolation of the 17th-century theory from its architectural roots and context in the first half of the 18th Century.

The "degeometrization of analysis" process saw the substitution of the vector definition, applicable to geometric objects, with the principle of feature as an algebraic method. This phenomenon was reflected in the seminal 1748 Introduction of Euler in Analysis Infinitorum, intended as a review of the principles and methods of research and analytical geometry essential for the examination of the calculus.

The Introduction of Euler was the first work of which the idea of purpose plays a specific and essential part. Euler states in the preface that mathematical research is the general science of variables and their purposes. Beginning with describing a function as "analytic language" (i.e., a "formula"):

A variable quantity feature is an abstract concept consisting of the variable quantity and amounts, or constant quantities.

Euler doesn't describe the word "analytic speech," but tries to give it sense by demonstrating that the four algebraic operations, roots, exponentials, logarithms, trigonometric functions, derivatives, and integrals are admissible "analytic expressions." He classifies algebraic or transcendental functions; single-valued or multivalued functions; and implied or concrete. The Introduction provides one of the earliest trigonometric equation treatments as integer ratios , as well as the first algorithmic logarithm treatment as exponents. Every method is algebraic.

While Euler did not begin with the notion of function, it was he who first brought it importance by approaching the calculus as a systematic theory of functions. As we can see, gradually the understanding of Euler 's functions was to develop.

Fourier and Fourier Series. Fourier 's research on heat conduction (submitted in 1807 to the Paris Academy of Sciences, but published in his seminal Analytic Theory of Heat only in 1822) was a pioneering phase in the development of the principle of motion. The major consequence of Fourier in 1822 was as follows.

Theory-Theorem. Every function described over is representable by a series of sines and cosines over this interval

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{\ell} + b_n \sin\frac{n\pi x}{\ell}\right],$$

where the coefficients a_n and b_n are given by

$$a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\cos\frac{n\pi t}{\ell}\,dt \quad \text{and} \quad b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\sin\frac{n\pi t}{\ell}\,dt.$$

Fourier 's declaration of this consequence met with skepticism. This disrupted many principles of mathematics in the 18th century. Euler and Lagrange (among others) were conscious of the effect, but only for those features. Fourier, of course, believed that it is so for all functions that the most common contemporary meaning was given to the word "function": the function usually represents a sequence of values or orders each of which is subjective. There is an infinite number of ordinates given to the abscissa x, which have real numerical values, either positive or negative or nil. We should not presume that these ordinates are subject to common law; they follow one another in some way whatsoever and each is provided as though it were a single quantity. Fourier's "proof" of his hypothesis was loose, even by early 19th-century norms. This was in essence formalism in the sense of the 18th century—"a play on representations in keeping with agreed principles but without any if little understanding of substance if purpose ". To persuade the skeptical mathematical world of his claim's reasonableness, Fourier had to explain this:

(a) Fourier series coefficients can be determined for any specified sequence
(b) Every function can be represented in (-l, l) by its Fourier series

CAUCHY

Mathematicians began formalizing all of the different branches of mathematics during the 19th century. One of the first to do so was Cauchy; his somewhat imprecise results were later made entirely rigorous by Weierstrass, who proposed developing calculus on arithmetic rather than geometry, favoring the definition of Euler over Leibniz's (see analysis arithmetic). Cauchy thought, according to Smithies, of functions as defined by equations involving real or complex numbers, and tacitly assumed that they were consistent:

In Chapter I, Section 1 of his Analyze algébrique (1821), Cauchy provides some general remarks regarding the roles. By what he states there, it is obvious whether he usually finds a function to be described by an abstract term (if explicit) or by an equation or an equation method (although implicit); although he varies by his predecessors, he is prepared to accept the probability that a function may only be specified for a restricted range of the exponential function.

LOBACHEVSKY AND DIRICHLET

Historically, Nikolai Lobachevsky and Peter Gustav Lejeune Dirichlet are credited with separately providing the current "formal" concept of a feature as a relationship in which the first element has a second unique element.

Lobachevsky (1834) states

The general definition of a function allows a function of x to be defined as a number given for every x and slowly varying with x. The function 's meaning may be provided either by an empirical term, or by a condition that provides a means of evaluating all numbers and choosing one of them; or finally the dependence can occur but remain unknown.

Though Dirichlet (1837) wrote

For now, there is a special finite y corresponding to each x, and moreover in such a way that if x differs continuously over the interval from a to b, then for this interval y is considered a continuous function of x. It is not appropriate here to give y in terms of x along the entire interval by one and the same rule, and it is not necessary to recognize it as a dependence articulated using mathematical operations.

Eves states that "in his introductory course in calculus, the student in mathematics typically follows the Dirichlet concept in structure. Imre Lakatos denied Dirichlet 's assertion to this formalization:

There can be no such definition at all in Dirichlet 's works, but there is ample evidence that he had no idea of this concept. In his [1837] paper, for example, when discussing piecewise continuous functions, he says that the function has two values at points of discontinuity: ...

However, Gardiner says, "...it seems to me that Lakatos goes too far, for instance, when he argues that 'there is sufficient proof that [Dirichlet] has little understanding of [the modern function] concept'." However, as mentioned above, Dirichlet's paper seems to provide a description along the lines of what is generally credited to him, even if (like Lobachevsky) he says it only for continuous function.

Likewise, Lavine observes:

It was a matter of some debate how much recognition Dirichlet merits for the current concept of a function, partially because he restricted his meaning to continuous functions .... I think Dirichlet established the notion of continuous function to make it plain that no regulation or law is essential except in the case of continuous functions, not only in general.

Since Lobachevsky and Dirichlet were regarded as being the first to incorporate the notion of arbitrary communication, this notion is often referred to as the Dirichlet or Lobachevsky-Dirichlet concept of a function. Bourbaki (1939) later used a generic variant of this description, and others in the educational world refer to it as the Dirichlet – Bourbaki concept of a function.

DEDEKIND

Dieudonné, who's one of the founding members of the Bourbaki party, credits Dedekind with his research Was sind und war sollen die Zahlen, a detailed and general modern description of a function, which emerged in 1888 but was drafted already in 1878. Dieudonné states that instead of being limited to simple (or complex) functions, as in previous definitions, Dedekind describes a function as a single-valued mapping between any two sets:

What was fresh and what was important for mathematics as a whole was the entirely general definition of a function.

CONCLUSION

Throughout all mathematics functions play an important role. They can be identified through graduate studies in any mathematics course beginning from pre-algebra. The origin of the concept now defined as feature has a long history that is only surpassed in depth by the several various forms it can be interpreted with functions. However, surface experience with definitions with such nature sometimes contributes to inconsistencies and misunderstandings. Researchers suppose that before new abstractions can be grasped, students must learn such concepts slowly and with careful attention to each level of understanding. Teachers and textbooks continue to give various contextual messages to pupils, and often pupils have disjointed expectations of roles after college attainment. Students sometimes built this from the instances they became more acquainted with, rather than the meanings they were taught. Although developmental factors preclude young people from completely understanding the term, the author claims that high school juniors can be faced with the idea of operation at its deepest degree of abstraction through careful and deliberate guidance.

REFERENCES

1. Ruthmae Sears, Dung Tran, Seoung Woo Lee, Amanda Thomas (2016) on Function Concept: Learning from History.

2. Annie Burns Childers, Draga Vidakovic (2014) on Students' Understanding of the Concept of Vertex of Quadratic Functions in Relation to Their Personal Meaning of the Concept of Vertex.

3. Lutfi Incikabi, Abdullah Çağrı Biber (2016) on Problems Posed by Prospective Elementary Mathematics Teachers in the Concept of Functions: An Analysis Based on SOLO Taxonomy.

4. Michiel Doorman, Paul Drijvers, Koeno Gravemeijer, Peter Boon, Helen L. Reed (2012) on TOOL USE AND THE DEVELOPMENT OF THE FUNCTION CONCEPT: FROM REPEATED CALCULATIONS TO FUNCTIONAL THINKING

5. Mustafa Gök, Abdulkadir Erdoğan, Emel Özdemir Erdoğan (2019) on Transpositions of Function Concept in Mathematics Curricula and Textbooks from the Historical Development Perspective.

6. Melanie M. Jones (2006) on Demystifying Functions: The Historical and Pedagogical Difficulties of the Concept of the Function.

7. Gyöngyi Szanyi The investigation of students' skills in the process of function concept creation.

8. Olov Viirman (2014) on The function concept and university mathematics teaching.

9. Mario Sánchez Aguilar, Rosa Isela González-Polo, Apolo Castaneda Research findings associated with the concept of function and their implementation in the design of mathematics textbooks tasks.

10. Rita Panaoura Michael, Chrysanthou Paraskevi, Andreas Philippou (2015) on Teaching the concept of function: Definition and problem solving.

11. Tamar Paz, Uri Leron (2009) on The Slippery Road from Actions on Objects to Functions and Variables.

12. Burton, L. (2004). Mathematicians as enquirers: Learning about learning mathematics.

13. Berger, M. (2013). Examining mathematical discourse to understand in-service teachers’ mathematical activities.

14. Viirman, O. (2012). The teaching of functions as a discursive practice: University mathematics teaching from a commognitive standpoint.

15. Viirman, O. (2013). The functions of function discourse.

16. Jablonka, E., Wagner, D., & Walshaw, M. (2013). Theories for studying social, political and cultural dimensions of mathematics education.

17. Tom Macintyre, Sheila Hamilton (2010) on Mathematics learners and mathematics textbooks: A question of identity? Whose curriculum? Whose mathematics?

18. M Carlson, M Oehrtman (2005) on Key aspects of knowing and learning the concept of function.