A Study of Harish-Chandra: A Kanpur Born Distinguished Mathematician of the World

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Abstract

According to R.P. Langlands [8] "Harish-Chandra was one of the outstanding mathematicians of his generation, an algebraist and analyst, and one of those responsible for transforming infinite-dimensional group representation theory from a modest topic on the periphery of mathematics and physics into a major field central to contemporary mathematics."

Keywords: Harish-Chandra Mathematics, Dirac matrices, tensor quantity, relativistic wave equation, Jacobi polynomial

Introduction

Prof. Harish-Chandra was born on October 11, 1923 in Kanpur in 'Sewa Sadan' near Lal Imli. His father's name was Shri Chandra Kishore Mehrotra and mother's name was Smt. Satyawati. He passed High School in 1937 from 'Christ Church High School' and Intermediate in 1939 from 'B.N.S.D. College', Kanpur. He received the B.Sc. degree in 1941 and M.Sc. degree in Physics from 'Allahabad University' in 1943. Sir C. V. Raman gave 100% marks in a written test for M.Sc. He spent the years 1943-45 at 'The Indian Institute of Science', Bangalore as J.H. Bhabha Memorial student under Prof. H.J. Bhabha. He went to Cambridge University as a research student of Nobel Prize winner Professor P.A.M. Dirac in 1945 and received his Ph.D. Degree from there in 1947 on "Infinite Irreducible Representations of the Lorentz Group". In the period 1947-49 he worked as research assistant of Prof. Dirac at 'Institute for Advanced Study', Princeton. In 1949-50 he was at 'Harvard University', Cambridge. He was at 'Columbia University', New York, from 1950 to 1963. In 1963 he became a permanent member of the 'Institute for Advanced Study' at Princeton, New Jersey and from 1970 onwards he had been the 'IBM Von Neumann' Professor there till his death on October 16, 1983.

He was married in 1952 with Lilitha Kale (Lily-Harish-Chandra). They have two daughters Pramila and Devika. He and his family lived in Princeton. Although he has not returned to India except for brief visits, he has always been profoundly concerned with India and Indian Science. He remains a figure of great inspiration in his native land.

He won Cole Prize of American Mathematical Society in 1954. He was Guggenheim Fellow from 1957 to 1958. In 1958 he was recommended for Field Medal (Prize in Mathematics equivalent to Noble Prize) but for some reasons prize was not awarded to him. He was Sloan Fellow from 1961 to 1963. He was elected a Fellow of the Royal Society (F.R.S.) in 1973. He was second F.R.S. in India after Ramanujan. He won Srinivas Ramanujan Medal of the Indian National Science Academy in 1974. He was elected Fellow of the Indian Academy of Science and of the Indian National Science Academy in 1975, and honorary fellow of T.I.F.R. He was member of the National Academy of Sciences of U. S. A. in 1981. He was awarded honorary doctorates by Delhi University in 1973 and by Yale University in 1981.

The whole work of Prof. Harish-Chandra is distributed in his 85 research papers [5] on theoretical physics, Representations of Lie groups and Lie Algebras, Character formulae and Plancherel theorem etc. contained in "Harish-Chandra Collected Papers" in Four Volumes edited by Prof. V.S. Varadarajan and published by Springer Verlag in 1984.

According to Prof. V.S. Varadarajan "Harish Chandra was the greatest Mathematician of Indian origin in our times". To pay a tribute to that greatest mathematician "Harish-Chandra Research Institute of Mathematics and Mathematical Physics", (H.R.I.), has been developed at Allahabad as an important centre of mathematical studies in India.

I am thankful to Dr. T.N. Trivedi (Retired Reader & Head, Deptt. of Mathematics, VSSD College, Kanpur) for introducing me to Harish-Chandra Mathematics. The studying of Harish-Chandra Mathematics is certainly interesting and is able to raise the mathematical level of the learner. I, therefore, like to suggest the students who want to develop in Mathematics should try to learn Harish-Chandra Mathematics.

Thoughts of Some Mathematicians

Prof. V.S. Varadarajan (University of California):

Prof. Varadarajan in [5] writes; "His work is a profound synthesis of algebra, geometry and analysis. The great force and continued resonance of his ideas have inspired a generation of mathematicians. The single-mindedness and courage with which he has pursued his goals, the beauty of his results, the power and originality of his methods, and the ultimate simplicity of his philosophy, as well as the conviction that sustains it, compel our admiration."

In [11] Prof. Varadarajan further writes; "Harish-Chandra was the greatest mathematician of Indian origin in our times. He began his scientific career as a theoretical physicist but gradually his interests became entirely mathematical, centering around the theory of representations of group. His ideas and the overwhelming force of his results transformed this subject into one of the central areas of mathematical activity today. By his capacity to discover beautiful things, by his prodigious ability to struggle towards distant goals undistracted by extraneous happenings and by his genius that was at all times leavened by a deep humility, he set an example that was inspiring to anyone who came into contact with him."

He finished his article [11] in the words; "To me he was not only a friend but a teacher in the fullest and most ancient sense of that world. It was from reading his papers that I discovered what serious mathematics was, and my countless conversations with him helped me grow as a mathematician as well as a person. Contrary to appearances he was a gentle and pure soul, very warm and very emotional. But a part of him he was always detached even from his own self. His letter to me consoling me on the occasion of my mother's death in 1979, written at a time when his own health was not very good, was typical of his attitude: caring, compassionate, yet detached. He was quite simply, the greatest and wisest man I ever knew."

Prof. Sigurdur Helgason (Massachusetts Institute of Technology):

Prof. Helgason writes [12]; "Developing theories for the so called universal enveloping algebra and the analytic vectors which now have became standard tools in representation theory, Harish-Chandra isolated and solved one basic problem after another; the solutions have became the pillars in the foundation of the subject. He quickly found himself in new unexplored territory and everywhere he looked there were natural conceptually compelling problems to deal with."

He further writes [12]; "With long hours of intense work, Harish-Chandra was an extremely prolific writer. He pursued his goals with firm determination, avoiding unnecessary distractions. Determined readers of his papers were, however, pleased to find them written with extreme clarity and care. A patient reader found no insurmountable obstacles because tedious details were given their due attention and not left to the reader to verify. This benefitted the subject of representation theory in two ways: workers in this field never hesitated using Harish-Chandra's results even if they had not worked through or even seen the proofs; Secondly, his uncompromising and precise style became an example for others to follow. I believe this has contributed in no small way to the health which representation theory has enjoyed in recent years."

Prof. G. Daniel Mostow (Yale University):

Prof. Mostow describes [12]; "I can describe the Harish of that period most accurately by quoting his own words. His first unforgettable remark occurred as we were walking back to the Institute after one of Chevalley's [2] lectures. The lectures were consistently polished with all details neatly in place, but on that particular day, Chevalley got stuck. Those of you who are familiar with Chevalley's uncompromisingly rigorous style may be amused to hear how Chevalley coped: he crouched against the blackboard, drew a diagram which he covered with his body, started at it finally erased the figure, and announced, "My assertion is certainly correct, but I do not see at the moment how to prove it." and he deferred the point for the following lecture. On our way back to the Institute, Harish declared with genuine puzzlement, "How can one know a mathematical statement is true without knowing how to prove it?"

The remark tells much about the young Harish. For him, to know a theorem was to know how to prove it. As he proved the impressive structure theorem after another about Lie algebras, he would have a publication-ready manuscript with no details spared as soon as he declared that he had a proof."

He further writes [12]; "I am a descendent of a religious tradition quite different from that of Harish. Nevertheless, I, too was stirred by the symbolism of Harish's ashes strew at this Institute. For as long as group representation theory continues to spurt with remarkable vitality, his spirit will hover here, and that will continue for a long, long time."

Prof. R.P. Langlands (Institute for Advanced Study, Princeton):

In [12] Prof. Langlands writes; "It is difficult to communicate the grandeur of Harish-Chandra's achievements and I have not tried to do so. The theory he created still stands - if I may be excused a clumsy smile-like a Gothic cathedral, heavily buttressed below but in spite of its grade weight, light and soaring in its upper reaches, coming as close to heaven as mathematics can. Harish, who was of a spiritual, even religious, cast and who liked to express himself in metaphors, vivid and compelling, did see, I believe, mathematics as mediating between man and what one can only call God. Occasionally on a stroll after a seminar, usually towards evening, he would express his feelings, his fine hands slightly upraised, his eyes intent on the distant sky; but he saw as his task not to bring men closer to God but God closer to men. For those who can understand his work and who accept that God has a mathematical side, he accomplished it."

Prof. Armand Borel (Institute for Advanced Study, Princeton):

On October 10, 1993 unveiling ceremony of bust of Prof. Harish-Chandra was held at M.R.I. Allahabad, there Prof. Armand Borel delivered a lecture [1]. At the end he delivered; "In mathematics, Harish life was indeed a search for fundamental general theorems, with the belief that they should be beautiful, and combine to harmonious theories. He pursued this quest with awesome single-mindedness, persistency, power and success". There are two papers jointly written by Prof. Harish-Chandra and Prof. Borel.

Prof. R.N. Ghosh (Allahabad University):

In 1950 Prof. Ghosh wrote [4]; "Einstein spoke of Madame Curie that she was one of those celebrated scientists whom fame had not corrupted. A similar statement was made by Dirac for Harish-Chandra, the brilliant Indian Physicist now working at Harvard University, that he is one of those few scientists who should come into dominative eminence in the Physics of the near future and whom physics expects to solve the various deadlocks of its problems."

The article of Prof. Ghosh includes an introduction which says; "Dr. Harish-Chandra is an old distinguished and loved student of Principal Khanna. After having won laurels at Cambridge University his is doing a recognized research work in Physics and Mathematics at the Universities of Princeton and Harvard in America. About him Sir C.V. Raman, greatest scientist of our country has predicted. "Given the full span of his life he might range with Newton, Euler and Lagrange." A short account of his career by his old teacher Dr. R.N. Ghosh, the head of the Physics Department of Allahabad University will interest the reader."

Dr. Pramila-Chandra (Daughter of Prof. Harish-Chandra):

The following paragraph taken from [9] contains a message of Prof. Harish-Chandra to his generations. The message awakens the power in a man.

"In a speech on October 10, 1993 in Allahabad, Harish-Chandra's elder daughter, Dr. Pramila Chandra, offered glimpses of her father: "A man of great vision and uncompromising standards – traits that made him both inspiring and impossible… He was a tough teacher and learning from him was not an easy process." If she went to him for assistance for homework he would ask, "Can you estimate the number of people who have successfully solved such questions in the last hundred years? Is it possible that you are dumber than all of them? And she would return to her desk "full of anger, pride and of course, determination." And when she returned at last with a solution, he would ask, "Is this the only way to solve this problem?" and would proceed to show a more elegant method. When sometimes in frustration she talked of giving up physics, he would respond, "If you can't understand something, you don't give up. You must think harder!"

Mathematics and Discussion

In fact the matrices are the representations of the operators corresponding to group elements. In paper [6] Prof. Harish-Chandra discussed the properties of Dirac matrices which arise in the relativistic wave equation of electron [3]. He has pointed out that when Dirac matrices are multiplied we get a group of order 64 having the centre $\{1,-1,i,-i\}$ and thus has a quotient group of order 16.
For the product of Dirac matrices he has established the formula:

$$E_{ab}E_{cd} = -\delta_{ac}\delta_{bd} + \delta_{bc}\delta_{ad} + E_{ac}\delta_{bd} - E_{bc}\delta_{ad} - E_{ad}\delta_{bc} + E_{bd}\delta_{ac} - \frac{i}{2}\varepsilon_{abcdef}E^{ef}, \tag{1}$$

here index a, b, c, d, e, f run from 0 to 5, δ_{ab} is the Kronecker delta function and ε_{abcdef} is an anti-symmetric tensor and $\varepsilon_{abcdef} = 1$.

To understand the analysis given in Harish-Chandra [6] it is essential to have an idea of such components of this tensor. For this purpose in [13] we have created a computer programme in 'C' which first obtains all the permutations of the set {0,1,2,3,4,5}, then arranges in increasing order, decides each whether it is an even or odd permutation and last prints the non-zero components of ε_{abcdef} in the increasing order of their arguments. The beauty of the paper [13] is output of C-program which is in a systematic way. We may extend this computer program to find all the permutations of degree n=6 to n=m, where m is greater than 6. The values obtained in [13] are useful in understanding the analysis of [6] and verifying the important tensor identities (39), (40) and others given in [6]. These tensor identities are

$$\varepsilon_{\alpha\beta\gamma\delta\lambda\rho}\,\varepsilon^{\lambda\rho\alpha'\beta'\gamma'\delta'}t_{\alpha'\beta'}t_{\gamma'\delta'} = 16(t_{\alpha\beta}t_{\gamma\delta} - t_{\alpha\gamma}t_{\beta\delta} - t_{\alpha\delta}t_{\gamma\beta}) \tag{2}$$

$$t_{\alpha\lambda}t_{\beta\rho}\varepsilon^{\alpha\beta\mu\nu\sigma\tau}t_{\mu\nu}t_{\sigma\tau} = \frac{1}{6}t_{\lambda\rho}\varepsilon^{\alpha\beta\mu\nu\sigma\tau}t_{\alpha\beta}t_{\mu\nu}t_{\sigma\tau} \tag{3}$$

With the applications of his beautiful formula (1) for the product of Dirac-matrices, Prof. Harish-Chandra evaluated the characteristic equation of a 4×4 matrix in his paper [6]. The analysis given in [14] is useful in studying the relativistic wave equation of an electron and gives a new method to find out the characteristic equation and determinant value of any 4×4 matrix in tensor form with the help of Harish-Chandra Formula (1). Using beautiful formula (1) characteristic equation of any 4×4 matrix T is given in tensor form as

$$\left\{(T-t)^2 + 2t_{\mu\nu}t^{\mu\nu}\right\}^2 - 8\left\{\left(t^{\alpha\beta}t_{\alpha\beta}\right)^2 - 2t^{\alpha\beta}t_{\beta\gamma}t^{\gamma\delta}t_{\delta\alpha}\right\}$$
$$-\frac{4}{3}it_{\alpha\beta}t_{\gamma\delta}t_{\lambda\rho}\varepsilon^{\alpha\beta\gamma\delta\lambda\rho}(T-t) = 0. \tag{4}$$

In [15] we have tried to go through paper [7] of Prof. Harish-Chandra. We have explained how the wave equation for the motion of an electron derived by Dirac [6], [7] has been transformed to the form suitable to the problem under consideration by Prof. Harish-Chandra. It is very interesting & much enjoying to throw light on the footstep directly written by him. We have given the details to verify the steps in the analysis used by him. The calculations have been given to arrive at the wave equation in the form:

$$\left[\frac{1}{i}\rho_1\left\{\sigma_3\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)+\frac{\sigma_1}{r}\left(\frac{\partial}{\partial\theta}-\frac{\sigma_3}{\sin\theta}\left\{M+\frac{n}{2}(1-\cos\theta)-\frac{\sigma_3}{2}\cos\theta\right\}\right)\right\}+\rho_3\mu+E\right]\psi_0 = 0 \tag{5}$$

The Jacobi polynomials [7] present on the process of solution of wave equation of electron moving in the field of a magnetic pole is not in the usual form looking in mathematical findings [10], [17]. We have compared both Jacobi polynomials in [16] and also given explicit form of $P^k_{m,j}(u)$ with the help of Harish-Chandra Identity (14) of [7] given as

$$\frac{\left(t_2\sin\frac{\theta}{2}+t_1\cos\frac{\theta}{2}\right)^{k-j}\left(t_2\cos\frac{\theta}{2}-t_1\sin\frac{\theta}{2}\right)^{k+j}}{\{(k-j)!(k+j)!\}^{\frac{1}{2}}}$$
$$=\sum_{m=k}^{-k}\frac{t_1^{k-m}t_2^{k+m}}{\{(k-m)!(k+m)!\}^{\frac{1}{2}}}P^k_{m,j}(\cos\theta) \tag{6}$$

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