MATHEMATICAL STUDY OF CONCENTRATION DISTRIBUTION IN PATHOGENIC TYPE CONTAMINATES FLUID FLOW THROUGH POROUS MEDIA

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Abstract

The concentration distribution in pathogenic type contaminates fluid flow through porous media is described by the governing equation of longitudinal dispersion phenomenon of one-dimensional concentration distribution in homogeneous porous media has been studied in term of advection-dispersion equation. This equation has been solved analytically by using integral transform, Duhamel's theorem along moving co-ordinates with suitable initial and boundary condition. The solution contains the terms of exponential and complementary error function. The analytical solution and its graph conclude that the concentration distribution of pathogenic type contaminates fluids decreases with time. The developed analytical solutions may support as a valuable tool for evaluating the aquifer concentration at any position and time.

Key words: Advection, dispersion, Integral transforms, Moving coordinates, Duhamel's theorem

Introduction

The recent days human being producing huge amount of waste water which is largely contains pathogenic type contaminates often create a hazard to the groundwater quality, Contaminating groundwater and its environment. Degradation of groundwater quality can occur in huge areas by miscible fluid flow through porous media from plane or diffuse sources like deep percolation from intensively farmed fields, or it can be caused by point sources such as mine spoils, septic tank, cemeteries, garbage disposal sites, and oil spoils or other unintentional entry of pollutants into the underground environment. Any more possibility is pollution by like seepage from polluted streams or intrusion of salt water.

The civilized human being contaminates the underground environment by dumping medical waste, chemical wastes and city garbage to dumping yard near water source like ponds, river or sea by without proper maintenance of pathogenic wastes. This yard creates the point source pollution of pathogenic or miscible type contaminant displacement of groundwater. This phenomenon in porous media plays an outstanding role in many science and engineering fields like the process of oil recovery in petroleum engineering, pollution of groundwater by waste product disposed underground movement of mineral in the soil and recovery of spent liquors in pulping process and many problems which involves flow in porous media.

The several researchers studied the analytical solution for one-dimensional advection dispersion equation in semi-infinite or finite porous media, mainly, Ogata and Banks [3] for a constant input concentration. Eneman et al. [15] gives the analysis for the systems where fresh water is overlain by water with a higher density in coastal delta areas. Meher [16] and Mehta [17] considered the Dispersion of Miscible fluid in semi infinite porous media with unsteady velocity distribution using a domain decomposing method An exact solution of the linear advection-dispersion transport equation with constant coefficients was introduced by Perez et al. [24] for both transient and steady state regimes and classic version of Generalized Integral Transform Technique was used in solving analytically. S R Sudheendra [26-28] Discussed the Mathematical Solutions of transport of pollutants through unsaturated porous media with adsorption in a finite domain and he gives the Mathematical Analysis of transport of pollutants through unsaturated porous media with adsorption and radioactive decay also gives the Mathematical Analysis of Solute Transport Exponentially Varies With Time In Unsaturated porous

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Mathematical Model

The one dimensional Advection-Dispersion equation describes the flow in homogeneous porous media along with initial and boundary conditions can be defined as

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2} - w\frac{\partial C}{\partial z} - \left(\frac{1-n}{n}\right)\frac{\partial S}{\partial t}.$$

The equilibrium isotherm between solution and adsorbed phase is given by $\frac{\partial S}{\partial t} = K_d\frac{\partial C}{\partial t}$, K_d is the distribution coefficient

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2} - w\frac{\partial C}{\partial z} - \left(\left(\frac{1-n}{n}\right)K_d\right)\frac{\partial C}{\partial t}.$$

$$\left(1 + \left(\frac{1-n}{n}\right)K_d\right)\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2} - w\frac{\partial C}{\partial z}.$$

$$R\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2} - w\frac{\partial C}{\partial z}.$$

Let us take $D_1 = \frac{D}{R}$ and $W_1 = \frac{w}{R}$ where $R = \left(1 + \left(\frac{1-n}{n}\right)K_d\right)$, initially, C=0, because saturated flow of fluid of concentration takes place in the porous media.

$$\frac{\partial C}{\partial t} = D_1\frac{\partial^2 C}{\partial z^2} - w_1\frac{\partial C}{\partial z}. \qquad (1)$$

Thus, the suitable boundary conditions for the given mathematical model

$$\left.\begin{aligned} &C(z,0) = 0 && z \geq 0 \\ &C(0,t) = C_0 e^{-\gamma t} && t \geq 0 \\ &C(\infty,t) = 0 && t \geq 0 \end{aligned}\right\} \qquad (2)$$

The problem then is to describe the concentration as a function of t and z. Where the input condition is assumed at the origin and a flux type homogeneous condition is assumed. Initial concentration is C0. To reduce equation (3) to a more familiar form, we considered

$$C(z,t) = \Gamma(z,t)\mathrm{Exp}\left[\frac{w_1 z}{2D_1} - \frac{w_1^2 t}{4D_1}\right]. \qquad (3)$$

Substitution of (3) reduces equation (1) to Fick's law of diffusion equation

$$\frac{\partial \Gamma}{\partial t} = D_1\frac{\partial^2 \Gamma}{\partial z^2}. \qquad (4)$$

Equation (4) is transformed by (1) and its initial and boundary conditions (5) are transformed by (2), the above transformations occur when we substitute equation (3) to equation (1) and its boundary conditions (2) respectively.

$$\left.\begin{aligned} &\Gamma(0,t) = C_0\, Exp\left[\frac{w_1^2 t}{4D_1} - \gamma t\right] && t \geq 0 \\ &\Gamma(z,0) = 0 && z \geq 0 \\ &\Gamma(\infty,t) = 0 && t \geq 0 \end{aligned}\right\} \qquad (5)$$

Differential Equation (4) may be solved. When a time dependent influx of the fluid at z = 0. The solution of equation (4) may be obtained readily by use of Duhamel's theorem. [Carslaw and Jeager, 1947].

If $\boldsymbol{C} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t})$ is the solution of the diffusion equation where media is semi-infinite with taking initial concentration is zero, its surface is maintained at concentration unity, then the solution of the problem in which the surface is maintained at temperature $\phi(t)$ is

$$C = \int_0^t \phi(\tau)\frac{\partial}{\partial t}F(x, y, z, t-\tau)d\tau.$$

This theorem is utilized principally for heat conduction problems, but the above has been specialized to fit this specific case of interest. Consider now the problem in which initial concentration is zero and the boundary is maintained at concentration unity. The boundary conditions are

$$\left.\begin{aligned} \Gamma(0,t) &= 0 \quad t \geq 0 \\ \Gamma(z,0) &= 1 \quad z \geq 0 \\ \Gamma(\infty,t) &= 0 \quad t \geq 0 \end{aligned}\right\}$$

When we Applying Laplace transform to equation (4) is transformed to an ordinary differential equation, that is,

$$\mathscr{L}\left[\frac{\partial \Gamma}{\partial t}\right] = \mathscr{L}\left[\mathrm{D}_1\frac{\partial^2 \Gamma}{\partial z^2}\right].$$

$$\frac{\partial^2 \overline{\Gamma}}{\partial z^2} = \frac{p}{\mathrm{D}_1}\overline{\Gamma}\,. \qquad (6)$$

Where $\overline{\Gamma}(z,p) = \int_0^\infty e^{-pt}\,\Gamma(\mathrm{z,t})\mathrm{dt}$ and p is the Laplace parameter. The solution of the above differential equation (6) is $\overline{\Gamma} = Ae^{-qz} + Be^{qz}$ where, $q = \pm\sqrt{\frac{p}{\mathrm{D}_1}}$ when we apply boundary condition as z = 0 we get $A = \frac{1}{p}$ and for the boundary condition as $z \to \infty$ we get B = 0 thus the particular solution of the Laplace transformed equation is

$$\overline{\Gamma} = \frac{1}{p}e^{-qz}$$

The inversion Laplace transforms of the above function is gives, the result

$$\Gamma = 1 - erf\left(\frac{z}{2\sqrt{\mathrm{D}_1 t}}\right) = \frac{2}{\sqrt{\pi}}\int_{\frac{z}{2\sqrt{Dt}}}^{\infty} e^{-\eta^2}d\eta.$$

Applying Duhamel's theorem, we get solution with the time dependent surface condition at z = 0 and initial concentration zero is

$$\Gamma = \int_0^t \phi(\tau)\frac{\partial}{\partial t}\left[\frac{2}{\sqrt{\pi}}\int_{\frac{z}{2\sqrt{D(t-\tau)}}}^{\infty} e^{-\eta^2}d\eta\right]d\tau.$$

We know that $\mathrm{e}^{-\eta^2}$ is a continuous function and we may differentiate under the integral, that is

$$\frac{2}{\sqrt{\pi}}\frac{\partial}{\partial t}\int_{\frac{z}{2\sqrt{D(t-\tau)}}}^{\infty} e^{-\eta^2}d\eta = \frac{z}{2\sqrt{\mathrm{D}_1 t}(t-\tau)^{3/2}}Exp\left[\frac{-z^2}{4\mathrm{D}_1(t-\tau)}\right].$$

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The solution is

$$\Gamma = \frac{z}{2\sqrt{\pi D_1}} \int_0^t \phi(\tau)\, Exp\left[\frac{-z^2}{4D_1(t-\tau)}\right] \frac{d\tau}{(t-\tau)^{3/2}}. \qquad (7)$$

Putting $\mu = \frac{z}{2\sqrt{D_1(t-\tau)}}$ then the equation (7) can be written as

$$\Gamma = \frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{Dt}}}^{\infty} \phi\left(t - \frac{z^2}{4D_1\,\mu^2}\right) e^{-\mu^2}\, d\,\mu\,. \qquad (8)$$

Since $\phi(t) = C_0\, Exp\left(\frac{w_1^2 t}{4D_1} - \gamma t\right)$, The particular solution may be written as

$$\Gamma(z,t) = \frac{2C_0}{\sqrt{\pi}} Exp\left(\frac{w_1^2 t}{4D_1} - \gamma t\right)\left\{\int_0^\infty Exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) d\mu - \int_0^\alpha Exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) d\mu\right\}. \qquad (9)$$

Where, $\alpha = \frac{z}{2\sqrt{D_1 t}}$ and $\epsilon = \sqrt{\frac{w_1^2}{4D_1} - \gamma}\ \left(\frac{z}{2\sqrt{D_1}}\right)$

Integrating integral part of the equation (9) the solution is

$$\Gamma(z,t) = \frac{2C_0}{\sqrt{\pi}} Exp\left(\frac{w_1^2 t}{4D_1} - \gamma t\right).\left[e^{2\varepsilon}\, erfc\left(\alpha + \frac{\varepsilon}{\alpha}\right) + e^{-2\varepsilon}\, erfc\left(\alpha - \frac{\varepsilon}{\alpha}\right)\right]. \qquad (10)$$

From equations (3) and (10)

$$\frac{c}{c_0} = \frac{1}{2} Exp\left[\frac{w_1 t}{2D_1} - \gamma t\right]\left[e^{2\varepsilon}\, erfc\left(\alpha + \frac{\varepsilon}{\alpha}\right) + e^{-2\varepsilon}\, erfc\left(\alpha - \frac{\varepsilon}{\alpha}\right)\right]. \qquad (11)$$

Re-substituting for α and ϵ in equation (11)

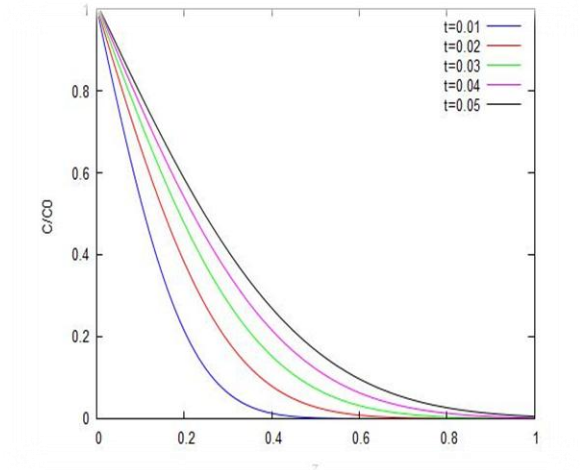
$$\frac{c}{c_0} = \frac{1}{2}\left\{ercf\left[\frac{z + t\sqrt{w_1^2 - 4D_1\gamma}}{2\sqrt{D_1 t}}\right] Exp\left[\frac{w_1 t - 2D_1\gamma t + z\sqrt{2w_1^2 - 4D_1\gamma}}{2D_1}\right] + ercf\left[\frac{z - t\sqrt{w_1^2 - 4D_1\gamma}}{2\sqrt{D_1 t}}\right] Exp\left[\frac{w_1 t - 2D_1\gamma t - z\sqrt{2w_1^2 - 4D_1\gamma}}{2D_1}\right]\right\}. \qquad (12)$$

Here $D_1 = \frac{D}{R}$, $W_1 = \frac{w}{R}$ and $R = \left(1 + \left(\frac{1-n}{n}\right) K_d\right)$.

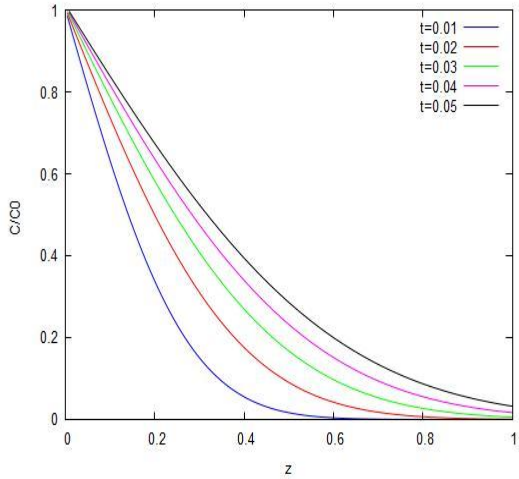
RESULTS and DISCUSSION:

The equation (12) represents concentration profile c/c_0 of the pathogenic type pollution, for any time t and distance z, the solution is obtained by using analytical method and We obtain the solution in terms of exponential form as well as complementary error function. The major constraints of the analytical methods are that the geometry of the problem must be regular. The applicability is for comparatively easy problems. This method is bit more fit than other standard methods for one dimensional transport problems. Following graphs represents the Break-through-curve for concentration profiles and distance along the porous media with porosity n. the ratio c/c_0 decreases with depth as n decreases the effect of distributive coefficient K_d, a velocity w and dispersion coefficient D whereas concentration profile versus time and for different values of depth z.

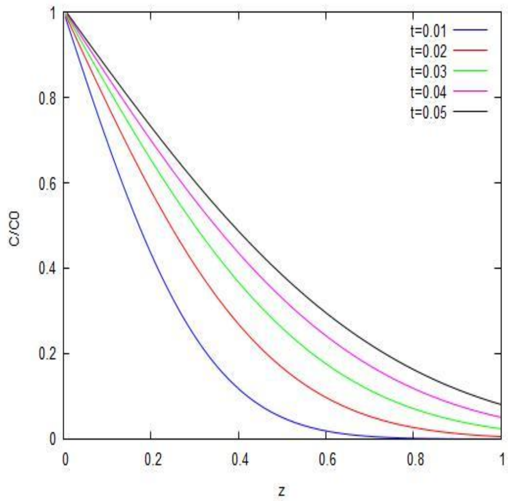
Figures 1 to 4 represents the concentration profiles verses time in the porous media for depth z. It is seen that for a different velocity w = 0.85m/day, w= 1.1 m/day w=1.35 m/day, w = 1.60 m/day with respect to the dispersion coefficient D = 1.3 m^2/day, D= 2.18 m^2/day, D = 3.28 m^2/day , D = 4.30 m^2/day and distribution coefficient K_d, C/C0 decreases with depth as porosity n decreases due to the distributive coefficient K_d and if time increases the concentration decreases for different time.



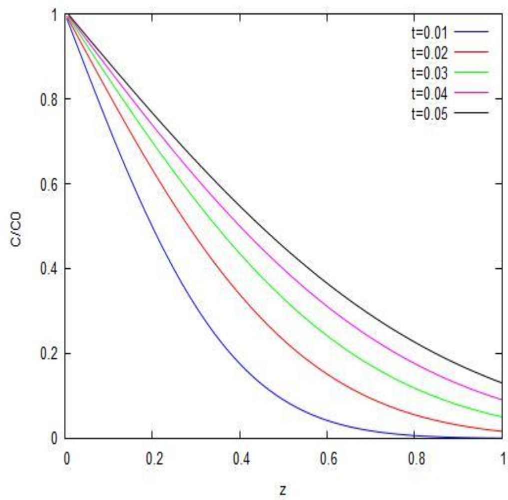
D=1.3 m^2/day & W=0.85 m/day



D=2.18m^2/day & W=1.1 m/day



D=3.28m^2/day & W=1.35 m/day



D=4.30m^2/day & W=1.60 m/day

CONCLUSION:

This mathematical model is dependable with physical phenomenon of the longitudinal dispersion of contaminated in homogenous porous medium. From the above graphs we can conclude that concentration of the salt water or contaminated is decrease as the distance z and time t increases. Laplace transform technique is a very useful method to obtain analytical solutions for solute transport in homogeneous porous media and under different flow atmosphere. The analytical expressions derived here are handy to the study of salinity intrusion in groundwater, useful in making quantitative predictions on the possible Contamination of groundwater supplies resulting from groundwater movement through buried wastes.

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