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A REVIEW ON USING CHAOS THEORY TO PREDICT DAILY TRAFFIC

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Abstract

Intercity road traffic volumes are typically calculated using probability functions, statistical methods, or meta-heuristic methods like artificial neural networks. Road traffic volumes are calculated using pattern recognition techniques since they depend on input variables such as the geometry of the road, the time of day or night, weekends or holidays, and other factors. The major goal of this research project is to evaluate how well chaos theory performs when used to estimate daily traffic volume utilising chaotic patterns. In this study, the daily traffic volume on intercity routes is analysed for chaotic behaviour, and the effectiveness of the chaos theory is reviewed and contrasted with probability functions. Data collected over the course of a year by installed automatic traffic counters were utilised in the analytical process to establish chaos factor as the ratio between the minimum and maximum daily traffic volume. The findings showed that daily traffic levels exhibit chaotic behaviour over a specified twenty-four-hour period. They also demonstrate that, for projecting daily traffic volume, the application of chaos theory is stronger than the uniform distribution function and inferior to the normal distribution function.

Keywords: Chaos theory, traffic volume estimation, probability function, pattern recognition tech-niques

1. Introduction

One of the most crucial considerations for traffic managers is estimating the volume of traffic, especially for bettering enforcement and management. For pavement design, fuel-tax revenue projections, and highway safety planning, data collection and daily traffic volume predictions are typically used. However, the expenses and personnel required for the monitoring efforts required for precise analytical average daily traffic (AADT) estimates are high [Rossi et al., 2012]. Traffic estimation is crucial for both tactical transportation needs like enforcement [Ashok, and Ben-Akiva, 2002] and for predicting the number and severity of road accidents [Ardekani, Hauer, and Jamei, 1995]. Transport authorities also put a lot of effort into estimating the volume of traffic on the roads in order to develop network-wide plans for enhancing enforcement and putting in place road safety measures. Traffic estimation has grown to be an increasingly important issue in road transport management in the era of adopting intelligent transportation systems developed around the globe to increase transportation efficiency [Caggiani et al. 2012]. While determining the origin-destination matrix is thought of as a fundamental element of transport planning, estimating road traffic volumes has grown in importance [Caggiani et al. 2012].

Many models and methodologies utilized for estimating traffic volumes are observed in the literature. The main techniques including Regression Analysis [Faghri and Hua, 1995], Ward's Minimum-Variance method of cluster-ing [Sharma and Werner, 1981], Clustering- based methods [Zhao, Li and Chow, 2004], and some heuristic methods of Genetic Algo- rithms [Lingras, 2001] and Artificial Neural Networks [Faghri and Hua, 1995, Lingras, 2001, Mahmoudabadi and Fakharian, 2010] have been implemented for road traffic esti- mation. Simulation techniques [Juran et al., 2009] and data mining [Gecchele et al. 2011] which are usually developed through using the probability distribution functions are also applied for traffic volume estimation. Varia- tion in day-to-day traffic volumes is a very important factor in the process of estimating daily traffic [Bodle, 1976].

On one hand, looking at the methods or tech- niques utilized for traffic estimation in the previous studies in detail send a notation to researchers that traffic volumes follow dy- namic patterns. On the other hand, chaos theory is known as a dynamic system, that will be discussed more in the second section, has been previously used for traffic or trans- port indicators. Therefore, it may be consid- ered as a major issue in road traffic modeling and proposing a dynamic pattern based on theconcept of chaos theory that may be a useful technique to develop a chaotic pattern for es- timating road traffic volumes. Consequently,

determining the performance of using chaos theory may convince researchers to develop chaotic patterns for road traffic estimation. Following the above mentioned issues, the ba- sic principle and the core of novelty of this re- search work is to consider daily traffic volume as a chaotic variable and develop an appro- priate model to predict daily traffic volume. While an appropriate chaos factor related to daily traffic volume is defined in the proposed methodology, the presence of chaos as a ma- jor concern for developing methodology will also be checked. It is assumed that, daily traf- fic volume is estimated in short-time predic- tion and the above concept may be useful for researchers and those who are interested to know whether chaos

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theory is a proper tech- nique to estimate daily traffic or not. Depend-ency upon initial condition is also considered as another concern to develop chaotic patterns [Mahmoudabadi and Seyedhosseini, 2012], sotraffic volumes are being estimated in short- time prediction for traffic management.

This paper is divided into seven sections. Af- ter introducing a short description about chaos theory, the paper is followed by methodology definition, where details of application of cha- os theory for daily traffic estimation are pre- sented. Details on case studies and analytical process are demonstrated after the above sec- tions, in sections 4 and 5, respectively. Discus- sions on rating the performances of three ap- proaches are presented in section 6, followed

by conclusion and some recommendations forfurther researches in the last section.

2. Chaos Theory

The concept of chaos theory was firstly in- troduced by Edvard Lorenz in 1963 [Gleick, 1987]. Chaos theory is known as a non-linear system, while in a chaotic nonlinear system, small differences in the initial conditions oc- casionally produce very great ones in the fi- nal outputs [Xh, Yb and Zhang, 2002]. It has been studied within the engineering scientific and mathematical communities and found to be useful in many disciplines such as high-performance circuits and devices, collapse prevention of power systems and also infor- mation processing [Mingjun and Huanwen, 2004]. Chaos theory has also been widely ap- plied in various fields of science, particularly in the area of traffic flow theory, commonly in short-time prediction, because of the exist- ence of the property of “sensitive dependence upon initial condition” [Lawrence, Feng and Huang, 2003].

In transportation systems, legal and social constraints may bind behavior, allowing re- searchers to predict human actions and their effects on system evolution, more accurately [Frazier and Kockelman, 2004, Sugihara and May, 1990]. In safety planning programming, it has also been approved that traffic measuressuch as the number of road accidents can be predictable under the property of chaotic be-

havior [Mahmoudabadi and Seyedhosseini,2012].

The largest Lyapunov exponent, usually indi-cated as λmax in the literature, is the clearest measure to prove the existence and to quan- tify chaos in a dynamic system or time series. Calculating the largest Lyapunov exponent is a more common technique to determine the presence of chaos that measures the conver- gence or divergence of the nearby trajectories[Frazier and Kockelman, 2004]. As the sys- tem evolves, the sum of series of values (in each dimension) will converge or diverge. Lyapunov exponents measure the rate of con-vergence or divergence in each dimension. Thus, if the largest Lyapunov exponent ex- ceeds zero, the system is chaotic [Frazier and Kockelman, 2004]. In other words, if the larg- est Lyapunov exponent is positive, it indicates that the system under investigation is sensi- tive to the initial condition and it is chaotic (Kiel and Elliott, 1996). Equation (1) is used to determine the largest Lyapunov exponent, where S(t) and S'(t) are the system situation and its nearest neighbor in time t, respectively. N is the number of available data points and Δt is time step or time span. Since the scientific background of chaos theory is related to math-ematical science, readers interested to know more about chaos theory are recommended to refer to [Gleick, 1987]. In this research work, the nearest neighbor for each situation is con-sidered as the situation on the previous time,

i.e. S'(t) = S(t – Δt). If Δt=1 means that time step is considered as a day and Δt=7 means time step is a week.

$$\lambda_{max} = 1/N\Delta t \sum_{(t=0)}^{(N-1)} Ln\left(|S(t+\Delta t)-S'(t+\Delta t)|/|S(t)-S'(t)|\right) \quad (1)$$

The well-known equation of logistic map is commonly applied to generate chaotic vari- ables [Sugihara and May, 1990; Trepaniera, Leroux, and Marcellis-Warin, 2009], par- ticularly in traffic behavior, because of good adeptness to time series. The logistic map equation is defined as equation (2) [Lawrence, Feng and Huang, 2003], in which P(t) is the chaotic variable in time 't', and K, is the equa-tion parameter which can be estimated by statistical techniques of mean square errors [Mahmoudabadi and Seyedhosseini, 2012] or pre-defined measure of 4 [Mingjun and Huan-wen, 2004].

$$P(t+1) = K \times P(t) \times (1-P(t)) \quad (2)$$

3. Methodology Definition

The proposed methodology is based on a pre- diction procedure following some stages. Ex-perimental data collected by automated elec- tronic systems are used for analytical process. In order to keep data accuracy, those gathered for more than 20 hours in a day, are used in the modeling and evaluation processes. Therefore, at the first stage, a filtering process is done

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Vol-10 Issue-3 No.01 March 2020

where data gathered for more than 20 hours a day are used in analytical process. Filter- ing stage is followed by calculating statistical measures of data in order to use for analytical process. It is necessary to define a proper cha-os factor, where results would be easily com-pared to other methods, so defining a chaos factor, which is the main part of contribution of this paper, is on the next stage, where a cha-otic variable is defined in closed interval [0 1]. According to daily traffic volumes (abbre-viated as TV), equation (3) is used to define chaos factor in the proposed model. For each part of time, known as (time span) or time period (t) represents a day, daily chaos factor (CF) is updated using logistic map equation defined as equation (4). Following the above steps, estimated daily traffic volume (ETV) for time span (t) will be obtained by equation

(5), where the maximum (MaxTVs) and mini- mum (MinTVs) of traffic volumes have been calculated by using experimental data.

$$CF(t)=(TV(t)-MinTVs)/(MaxTVs-MinTVs) \quad (3)$$

$$CF(t+1)=K\times CF(t)\times(1-CF(t)) \quad (4)$$

$$ETV(t+1)=MinTVs+(MaxTVs-MinTVs)\times CF(t+1) \quad (5)$$

Two approaches are utilized for validation process, where criteria measures for accura- cy are defined based on the whole data and the last five and fifteen days. Checking accu- racy for the mentioned categories is related to checking the methodology performance in both long-term and short-term predictions. Checking the performance of proposed meth- odology and comparing results follows as the final stage. An overall view of the proposed methodology is demonstrated in figure 1.

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

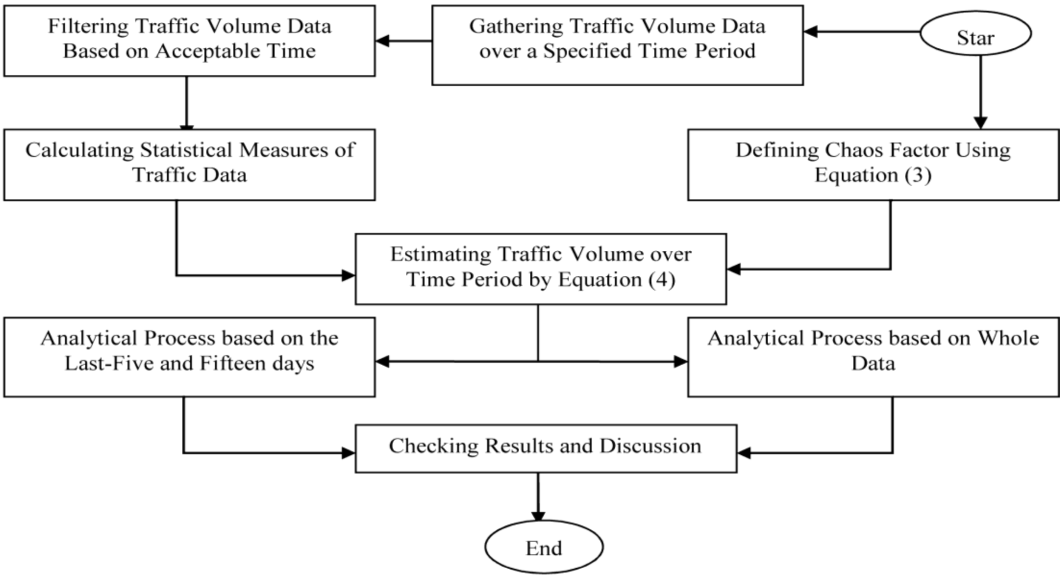

Figure 1. Overall view of the proposed methodology

4. Case Study and Experimental Data Two intercity roads of Qazvin-Rasht andTabriz-Soufian have been selected as casestudies. Qazvin-Rasht is a freeway, 130 kmlength and two lanes in each direction. Tabriz-Soufian is a divided road, 50 km length withtwo lanes in each direction. The difference be-tween the above roads is due to the existenceof U-Turns. One of the main issues on select-ing the above roads as case studies is to selectroads in different topographic areas, where Qazvin-Rasht is located in the mountainouslocation and Tabriz-Soufian is located in aflat area. The other issue is data availabilityover a year, while reliable data have been col-lected by the selected system according to lo-cal experts' point of views. There are U-Turnaccesses in Tabriz-Soufian road, so Qazvin-Rasht and Tabriz-Soufian are known Freewayand Express ways, respectively, according tonational instructions. Automatic traffic coun-ters are installed in the surface of pavementin several parts of the above selected roadswhich collect data on traffic volumes and alsospeed detection systems through inductionloops. Systems detect the vehicles passingon the surfaces, recognize the axle load pat-terns and eventually categorize the vehicles according to the specified axle configurations.Installed automatic traffic counters are able torecord traffic data whereas each record con-sists of date, time, location ID, axle configu-ration pattern and speed, which is automati-cally calculated according to the times when

vehicles are detected at the beginning and end of installed loops, but the pack of data is transferred every 15 minutes which are appro-priate to be studied in the various researches. Data over a year started from 21-March 2012 to 20-March 2013, being collected through electronic devices installed for the purpose of traffic counting transformed in the form of daily traffic volumes. The accuracy of traffic counting results is an essential concern, so data can be

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Vol-10 Issue-3 No.01 March 2020

collected in daily traffic volume pattern, while those with less than 20 hours ofgathering data have not been used for analyti-cal process. It is obvious that the accuracy of validation process will be improved if experi-mental daily traffic volumes of 24 hours are used in the process of validation, but the num-ber of available data may hamper to achieve the best results. Therefore, the best decision has been made on using daily traffic volumesof available data with more than 20 hours of full collecting data.

5. Analytical Process

Drawing bar charts which represent a schemat-ic pattern of frequency of daily traffic volumeover a year is the first step of analytical pro- cess. This can help decision makers to imag- ine an overall perspective for obtaining proper distribution functions. Hence, two bar charts based on the minimum and maximum of traf-fic volumes in the selected roads are drawn infigures 2 and 3. In addition, statistical meas- ures of two data sets will help decision mak-

ers to develop better distribution functions for all data sets. Daily traffic volumes have been analyzed and statistical measures includingminimum, maximum and standard deviation

are presented in table 1.

The second step of analytical process is to check the presence of chaos. According to thesection of methodology definition, chaos fac-

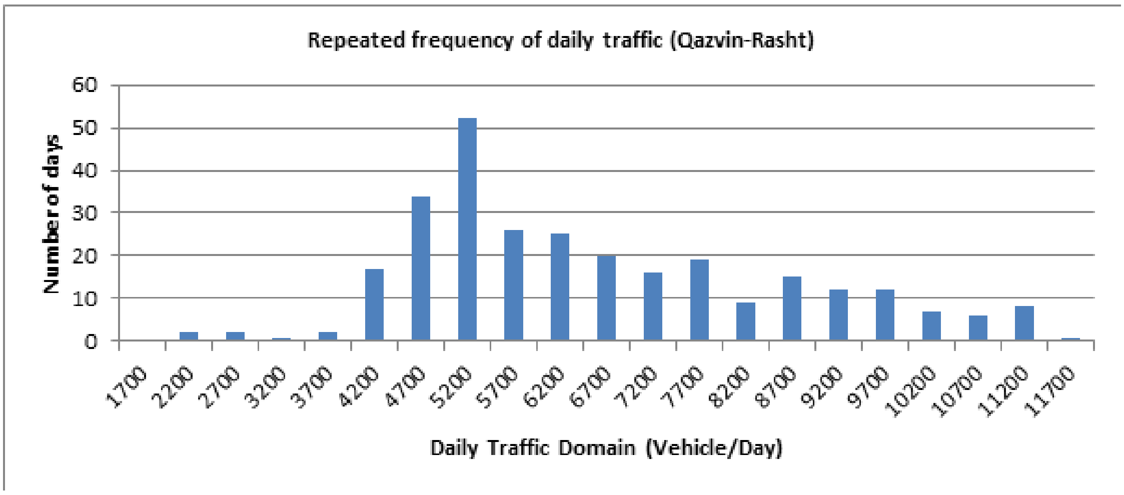

Figure 2. Bar Chart of the Frequency of Daily Traffic Volume for Qazvin-Rasht Freeway

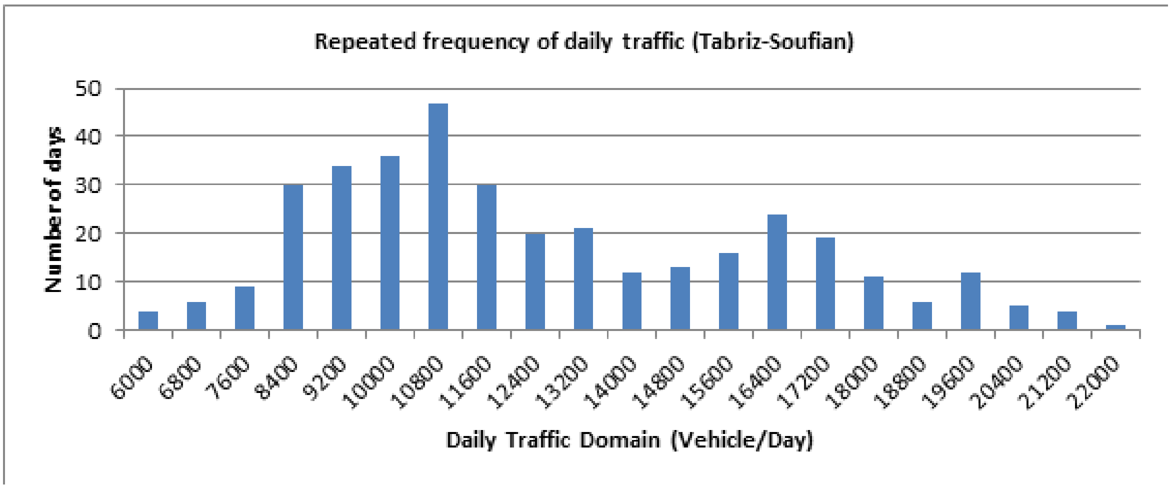

Figure 3. Bar Chart of the Frequency of Daily Traffic Volume for Tabriz-SoufianExpress Way

Table 1. Statistical measures for daily traffic volume

Road	Number of records	Min	Max	Mean	Standard Deviation	Largest Lyapunov Exponent ($\times 10^{-3}$)	
						Δt=1	Δt=7
Qazvin-Rasht	346	1765	41045	8599	6006	4.37	-5.96
Tabriz-Soufian	363	2250	22415	12217	3775	6.11	1.24

tor is the ratio between the maximum and theminimum of traffic volumes over a year. Equa- tion (3) is used for calculating chaos factor.In this case the maximum of chaos factor is calculated as 1 for day 73 (with traffic volume 41045) and calculated as 0 for day 283 (with traffic volume 1765) in figure 4 corresponds to data gathered in Qazvin-Rasht. They have also been calculated for days 162 and 249 in figure 5 which corresponds to data gathered in Tabriz-Soufian. The variation of chaos factorsfor two data sets is drawn and shown in fig- ures 4 and 5 corresponding to Qazvin-Rasht and Tabriz-Soufian, respectively. Numerical

results are tabulated in table 1. As the last col- umns of table 1 revealed, the behavior of daily traffic volumes for both roads are chaotic, but weekly traffic patterns are not. The main rea- son is that the weekly patterns are the same forroad traffic volumes over a year, because the Largest Lyapunov Exponent for daily traffic patterns are greater than 0 but they are not for weekly traffic patterns.

6. Discussion

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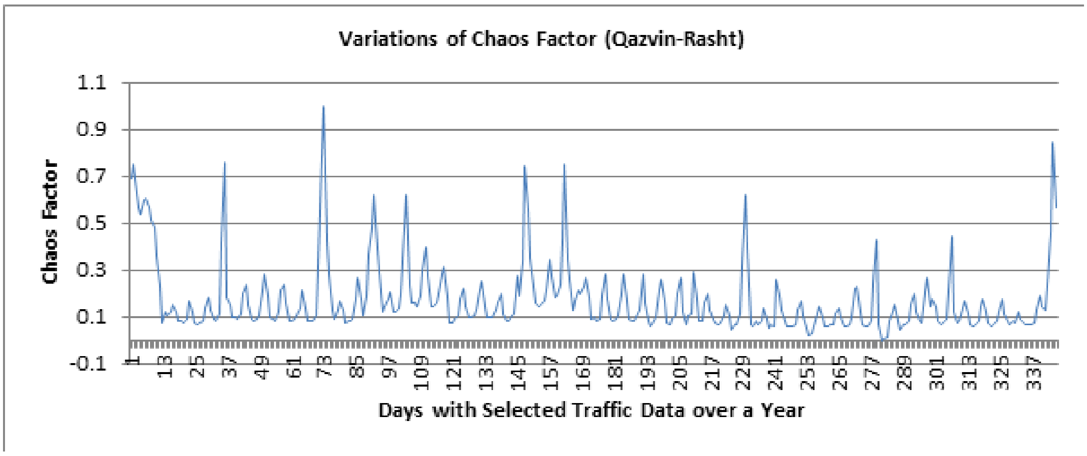
Vol-10 Issue-3 No.01 March 2020

In order to recognize the probability function, two histograms for traffic data gathered over a year outlined in the previous section have



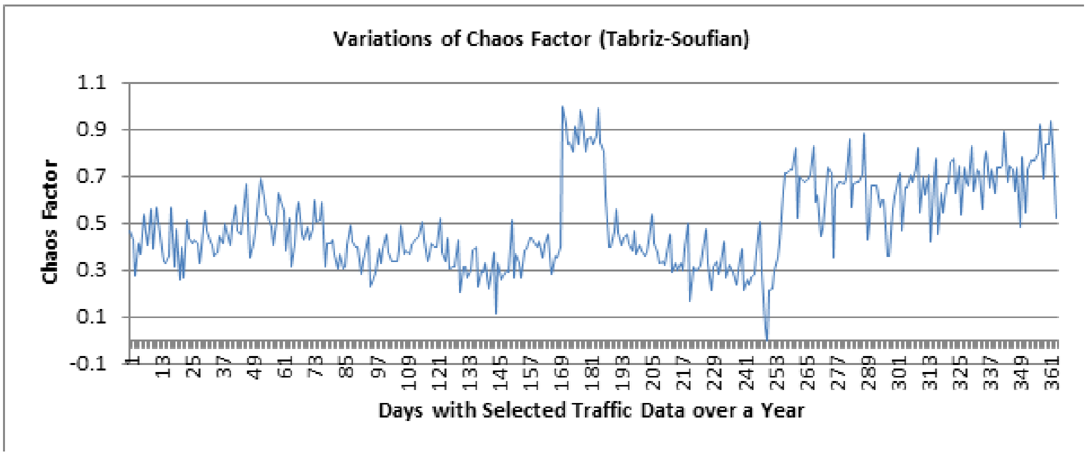


Figure 4. Variation of Chaos Factor for Qazvin-Rasht Freeway over a Year

Figure 5. Variation of Chaos Factor for Tabriz-Soufian Expressway over a Year

been analyzed and it seems that two different distribution functions of normal and uniform may be compatible to traffic data sets and needs to be checked. Therefore, two different probability functions of normal and uniform have been utilized to estimate traffic volume and compare results with using chaos theory applied in this research work. Following that, two criteria measures of mean square errors and difference between the number of ob-servation and model output in each domain [Mahmoudabadi and Ghazizadeh, 2013] are considered for validation. Domains are de- fined for each data set regarding to minimum and maximum daily traffic volume over a year. The mean square errors corresponding to three utilized time prediction period are shown in table 2. The above periods used in analytical process show the number of daily traffic estimations in which 5 days, 15 days and the whole data set are used to calculate

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Vol-10 Issue-3 No.01 March 2020

mean square errors.

As shown in table 3, the best results are re- lated to normal distribution function, where the minimum of mean square errors between

model outputs and observations are obtained. Chaos theory is rated in the second, in which criteria measures are better than uniform dis- tribution function and weaker than normalone.

Results for all domains are also shown in ta- bles 3 and 4. In this method, difference be- tween observations in each domain is com- pared to the number of output model. Tables revealed that the number of differences indomains have the same conclusion on rating the performances of modeling, whereas the average differences between observations andmodels' outputs are set in the order of normal,chaos and uniform.

7. Summary and Conclusion

In order to assess the performance of applyingchaos theory for daily traffic estimation, two main roads of Qazvin-Rasht and Tabriz-Sou- fian have been selected. Experimental daily traffic data, gathered through automatic traf- fic counter, have been analyzed for checking the performances of utilizing normal and uni-form distribution functions versus applying

Table 2. Mean square errors of data and models' outputs (×106)

Time period of prediction	Qazvin- Rasht			Tabriz- Soufian		
	Normal	Uniform	Chaos	Normal	Uniform	Chaos
5-day estimation	267	659	309	31	126	38
15-day estimation	122	568	400	40	99	61
Total error	68	330	374	29	44	63
Performance rank	First	Third	Second	First	Third	Second

Table 3. Difference between the number of observation and model output (Qazvin-Rasht)

Domain		Real	Normal		Uniform		Chaos	
Lower	Upper	No.	No.	Diff.	No.	Diff.	No.	Diff.
	<1700	0	31	31	0	0	14	14
1700	2200	2	4	2	4	2	15	13
2200	2700	2	4	2	5	3	9	7
2700	3200	1	17	16	5	4	8	7
3200	3700	2	12	10	3	1	11	9
3700	4200	17	12	5	5	12	9	8
4200	4700	34	7	27	8	26	6	28
4700	5200	52	13	39	4	48	4	48
5200	5700	26	11	15	4	22	3	23
5700	6200	25	10	15	4	21	2	23
6200	6700	20	15	5	6	14	5	15
6700	7200	16	12	4	7	9	4	12
7200	7700	19	6	13	3	16	4	15
7700	8200	9	8	1	4	5	4	5
8200	8700	15	15	0	4	11	4	11
8700	9200	12	19	7	3	9	5	7
9200	9700	12	13	1	5	7	6	6
9700	10200	7	10	3	5	2	0	7
10200	10700	6	9	3	2	4	3	3
10700	11200	8	13	5	3	5	6	2
>11200		1	11	10	4	3	224	223
Average				10.19		10.67		23.14

Table 4. Difference between the number of observation and model output (Tabriz-Soufian)

Domain		Real	Normal		Uniform		Chaos	
Lower	Upper	No.	No.	Diff.	No.	Diff.	No.	Diff.
	<6000	4	13	9	65	61	108	104
6000	6800	6	15	9	11	5	9	3
6800	7600	9	13	4	13	4	13	4
7600	8400	30	12	18	13	17	9	21
8400	9200	34	22	12	13	21	7	27
9200	10000	36	18	18	10	26	8	28
10000	10800	47	30	17	14	33	9	38
10800	11600	30	28	2	18	12	9	21
11600	12400	20	29	9	24	4	6	14
12400	13200	21	27	6	16	5	13	8
13200	14000	12	28	16	15	3	12	0
14000	14800	13	32	19	15	2	10	3
14800	15600	16	21	5	13	3	7	9
15600	16400	24	27	3	13	11	12	12
16400	17200	19	17	2	13	6	12	7
17200	18000	11	9	2	11	0	12	1
18000	18800	6	6	0	7	1	17	11
18800	19600	12	6	6	20	8	11	1
19600	20400	5	4	1	27	22	10	5
20400	21200	4	3	1	9	5	18	14
>21200		1	0	1	14	13	51	50
Average				7.62		12.48		18.14

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Vol-10 Issue-3 No.01 March 2020

chaos theory. The ratio between the minimum and maximum traffic volumes over a year is considered as chaos factor. Following that the presence of chaos has been checked by calcu- lating the largest Lyaponuv exponent. At the first step, results showed that the daily varia- tion for road traffic can be considered as cha- otic pattern, but applying chaos theory is not an appropriate technique for estimating daily traffic volumes compared to normal distribu- tion function. Applying three approaches of using normal and uniform distribution func- tions as well as the concept of chaos theory revealed that the performances of three ap- proaches are ordered as normal distribution function, chaos theory and eventually uniform distribution function. The main conclusion ofthis research work lies on the topic that al- though daily traffic volumes may adapt with chaotic behavioral patterns but the best way isto check the reliable data probability functions when there is no input variables for modeling traffic volume recognition. In other words,the main result of this research work for re- searchers interested on using chaos theory forestimating daily traffic is to use proper distri-bution function instead of chaos theory, but because of existing dynamic patterns it wouldbe better to be utilized instead of uniform dis- tribution function.

Researchers, interested on studying in this topic, are recommended to search for another definition of chaos factors for road traffic pat-terns. The other parameters of traffic may be found to have chaotic patterns. They are also recommended to check the power of chaos- based techniques compare to the other statis- tical techniques such as regression models, while input parameters are available.

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