Trigonometric Shear Deformation Theory for Thermal Flexural Analysis of Isotropic Plate

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Abstract

When structures are exposed to a harsh thermal environment, which causes significant thermal stresses to form in them, isotropic plates are used very successfully. The correct thermal response of plates as well as the consequences of shear deformation call for sophisticated theories. The displacement field in use has three variables. The trigonometric sine function is utilised in the displacement field in terms of thickness coordinate to describe the impact of shear deformation. One benefit of theory is that the constitutive laws can be used to directly calculate the transverse shear stresses, negating the need for a shear correction factor. Transverse shear stresses are distributed parabolically along the thickness of the plate as well as at its top and bottom surfaces, providing the requirements for a stress-free boundary. The theory's boundary conditions and governing differential equations are obtained using the principle of virtual work. The findings of the bending analysis of isotropic plates that are subjected to heat load are compared with other theories to verify the accuracy of the current theory.

*Keywords:D*isplacement field; isotropic beam; simply supported plate; thermal load

1. Introduction

This study develops a variationally consistent trigonometric shear deformation theory for generated heat loads. To accurately derive the governing differential equations and boundary conditions, the principle of virtual work is applied. With the aid of governing differential equations, the stiffness matrix is used to build the solution for the outfield variables w,, and. The theory is used to analyse the thermal behaviour of a solid rectangular plate that is uniformly supported and isotropic. We obtain both general and closed-form answers. The generic solutions for the variables w,, and are derived for the plate under consideration using appropriate boundary conditions. Obtained results are compared with those of higher order shear deformation theory and classical plate theory.

The eminence of the present theory is acclaimed by precise evaluation of thermal stresses and displacements. The theoretical formulation of a uniform plate is obtained by taking in to account the certain kinematical and physical assumptions. The dynamic version of the principle of virtual work is referred to obtain the variationally correct forms of differential equations and boundary conditions, based on assumed displacement field. In order to overcome the limitations of FSDT, higher- order shear deformation theories (HSDTs) involving higher-order terms in Taylor's expansions of the displacements in the thickness coordinate were developed by Reddy [1], Ren [2] and Kant and Pandya [3]. Ghugal and Kulkarni [4] have used trigonometric shear deformation theory (TSDT) for thermal analysis of composite plates. Shinde and Kawade [5] have presented thermal response of isotropic plates using hyperbolic shear deformation theory. Shimpi [6] created the refined plate theory (RPT) for isotropic plates, which has two variables and just two unknown functions. Ghugal and Shimpi [7] gave a thorough analysis of plate theories for isotropic and laminated plates, whereas Thai and Kim [8] reported on functionally graded plates. Reddy [9] provides the philosophy needed to comprehend shell theory and the physics of laminated composite plates. Isotropic plate thermoelastic stress analysis was presented by Boley and Weiner [10].

The plate under consideration

The principle of virtual work is used to obtain the variationally correct forms of differential equations and boundary conditions, based on the assumed displacement field. The occupied region of the plate is: Consider a thick isotropic simply supported plate of length a in x direction, Width b in y direction and depth h in z direction. Where x, y and z are Cartesian coordinates. The whole length of plate is subjected to thermal load of intensity $T(x)$. The axial displacement, transverse displacement, axial bending stress and transverse shear stress are required to be found out under this condition. The plate material obeys the generalized Hook's law. The plate is made up of homogeneous linearly elastic isotropic material with the principal material axes parallel to the x and y axes in the plane of plate.

The displacement field of TSDT

The following is the expression for the displacement field. The trigonometric function is assigned according to the shearing stress distribution through the thickness of plate

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

$$u(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} + \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)\phi(x,y) \qquad (1)$$

$$v(x,y,z) = -z\frac{\partial w(x,y)}{\partial y} + \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)\psi(x,y)$$

$$w(x,y,z) = w(x,y) \qquad (2)$$

Where u and v are the in-plane displacement components in the x and y directions respectively, and the transverse displacement in the z direction is w. The trigonometric function in terms of the thickness coordinate in both the in-plane displacements u and v is associated with the transverse shear stress distribution through the thickness of plate. The functions ϕ (x, y) and ψ (x, y) are the unknown functions associated with the shear slopes.

Governing equations and boundary conditions

Principle of virtual work is used to obtain the governing differential equations and boundary conditions. Variationally consistent differential equations for plate under consideration are obtained by using and solving the equations for stresses, strains and principle of virtual work. The principle of virtual work when applied to plate leads to:

$$\int_{x=0}^{x=a}\int_{y=0}^{y=b}\int_{z=-h/2}^{z=h/2}\left(\sigma_x\,\delta\varepsilon_x + \sigma_y\,\delta\varepsilon_y + \tau_{yz}\,\delta\gamma_{yz} + \tau_{zx}\,\delta\gamma_{zx} + \tau_{xy}\,\delta\gamma_{xy}\right)dx\,dy\,dz - \int_{x=0}^{x=a}\int_{y=0}^{y=b} q(x,y)\,\delta w\,dx\,dy = 0 \qquad (3)$$

 ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

∫ ∫ ∫ σ δε + σ δε + τ δε + τ δε + τ δε

x 0

Where δ = variational operator,

Substituting expressions for the strains and stresses in Equation (3) and employing Green's theorem successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the plate. The obtained governing differential equations are as follows:

$w: D_{11}$

$4\ w$

4

$2\ D_{12}\ 2D_{66}$

$4\ w$

$2\quad 2\quad D_{22}$

$4\ w$

4

S_{11}

3

S_{22}

$3\ 3$

$3\quad S_{12}\quad 2S_{66}$ ( )

3

$2\ 3$

2

x

$^2T_1$

$x\quad x$

y

$^2T_1$

x

$x\qquad x\ y$

$x\quad y$

$TD_{11}\quad TTD_{12}$

x^2

$TD_{12}\quad TTD_{22}$

$y^2\quad q,$

(4)

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

$\partial^3 w$

$2 \quad \partial^3 w$

$\partial^2 \quad 2$

∂T_1

$: S_{113} \quad S_{12} \quad 2S_{66}$

$2 \quad SS_{11}$

$2 \quad SS_{66}$

$2 \quad C_{55} \quad SS_{12} \quad SS_{66}$

$TS_{11} \quad TTS_{12}$

$0, (5)$

∂x

$\partial^3 w$

$\partial x \partial y$

$\partial^3 w$

∂x

2

∂y

∂^2

$\partial y \partial x$

∂^2

∂x

∂T_1

$: S_{22}$

$3 \quad S_{12} \quad 2S_{66} \quad 2 \quad SS_{66}$

$2 \quad SS_{22}$

$2 \quad C_{44} \quad SS_{12} \quad SS_{66}$

$TS_{12} \quad TTS_{22}$

$0, (6)$

$\partial y \quad \partial y \partial x \quad \partial x$

∂y

$\partial y \partial x \quad \partial y$

The obtained associate consistent boundary conditions are as below:

Along the edge $x = 0$ and $x = a$,

$\partial^3 w$

$\partial^3 w \quad 2 \quad 2$

D_{22}

$3 \quad D_{12} \quad 4D_{66}$

2 □ □ $2S_{66}$
2 □ $S_{66}2$ □

□y

□ 2□

□y□x □ □x

□T_1

□y □

□ □ S_{12} □ $2S_{66}$ □

□ □TD_{12} □ TTD_{22} □
□y□x □y

□ 0 or w is prescribed

(7)

□
□ D_{12}

□^{2} w

2

□ D_{22}

□^{2} w □ □ □
2 □ □ S_{12}
□ □x

□ □

□y □ □x

□w

□S_{22}

+ □TD_{12} □ TTD_{22} □T_1 □ 0 or is prescribed
□y □x

(8)

SS_{66} □ □ □ □

□ □ □ □
□ □ □ $2S_{66}$

□ ^{2} w
□ 0

or □ is prescribed (9)

□□y
□2 w
□x □

□□

□y□x
□2 w □□

S_{12}

2 □ SS_{12}
□x

□ S_{22}
□x

2 □ SS_{22}
□y

□ TS_{12} □ TTS_{22} □ T_1 □ 0 or □ is prescribed

(10)

□y

Along the edge y □ 0 and x □ b,

□3 w

□3 w □
□ 2□ □ 2□ □

□ D_{11}

3 □ □D_{12} □ 4D_{66} □

2 □ □ 2S_{66}
2 □ S_{662} □

□x

□2□

□x□y □ □y

□T_1

□x □□ □ S_{12} □ 2S_{66} □

□ □TD_{12} □ TTD_{22} □
□y□x □x

□ 0 or w is prescribed

(11)

□
□ D_{12}

□2 w

2

□ D_{22}

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

$^{2}w \quad {}_{2} \quad S_{11}$

S_{12}

$+ \; TD_{12} \; TTD_{22} \; T_1 \; 0$ $\frac{\partial w}{}$ or is prescribed (12)

$x \quad y \quad x \quad y \quad x$

S_{11}

^{2}w

2

S_{12}

$^{2}w \quad 2 \quad SS_{11}$

SS_{12}

$- \; TS_{11} \; TTS_{12} \; T_1 \; 0$ or is prescribed (13)

x

$y \quad x \quad y$

^{2}w

SS_{66}

x

$2S_{66}$

y

0

$x y$

or is prescribed (14)

The flexural behaviour of the plate is described by the solution satisfying these equations and the associated boundaryconditions at each edge of the plate.

The general solution of governing equilibrium equations of the plate:

The following is the solution form for $w(x, y)$, $\phi\,(x, y)$ and $\psi\,(x, y)$ satisfying the boundary conditions givenby the equations through perfectly for the plate with all the edges simply supported.

$y \qquad m \; x$

$w \; x, y \quad w_{mn} \sin \; \sin_{a} \; a$ cos b

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

m 1 n 1

m x

n y

m 1 n 1

m x

n y

T1 x, y mn cos sin sin
Tmn sin a
b
m 1 n 1 a b

Where *wmn*, *ϕmn*, and *ψmn* are the unknown coefficients which can be easily determined by substituting above Equations and *T(x, y)* = *zT1(x, y)* in the set of three governing differential equation and solving the resulting simultaneous equation as

$K_{11}w_{mn}$ K_{12} mn K_{13} mn F_1 $K_{21}w_{mn}$ K_{22} mn K_{23} mn F_2

$K_{31}w_{mn}$ K_{32} mn K_{33} mn F_3

n^2 2

m^2 2

(15)

F_1 D_{12} x D_{22} y

2 T_{1mn} D_{11} x D_{12} y

2 T_{1mn},

b a

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

2 F_{11} S

S_x 12

m T

$, F$ S S

n T_y $1mn$ 3 12 x 22 y $1mn$
a b

Illustrative Example

In order to prove the efficiency of the present theory, the simply supported isotropic rectangular plate *(a = 1.5b)* considered, having material properties as,

Modulus of Elasticity E = 210 GPa

Poisson's Ratio μ = 0.30

Coefficient of Thermal Expansion = = $12X10^{-6}$/°C

2. Results and Discussion

Numerical Calculation

The results for maximum transverse displacement , in-plane displacements and , in-plane normal stress components and , the in-plane shear stress components transverse shear stress components and are presented in the following non dimensional form for the purpose of presenting the results in this work.

u 1 2 u b 0, h ,
$1 T_0 E_2 b$ 2 2
xy
$1 T_0 E_2 b^2$
xy 0, 0,
, 2
v 1 2 v a , 0, h ,
10 b
xy $1 T_0 E_2 b$ 2 2

2 □ □ xz □ □ 0, , 0 □ ,

10 □ a b h □
w □ □ □ 1 T0 E2 b □ w2 □ ,, □ 2 □ ,
2
10 □ a □
□ □ □ 1 T0 E2 b □ □
( ) 1
□ 22 2 □
yz □ a b h □
2
□ yz □ □ □ 1 T0 E2 b □ 2
, 0, 0 □ ;
2 □ x, □ xy, □ y □ □

□ □ 1 T0 E2 b □
□ 22 2 □

All the parameters are obtained by solving the force matrix and equilibrium equations.

Table 2.1: Maximum transverse displacement □ at *(x = a/2* and *y = b/2)*, in-plane displacement components □ and □ at *(x = 0, y = b/2* and *z = h/2)* and *(x = a/2, y = 0* and *z = h/2)* respectively, normal stress components □ □ and □ □ at *(x = a/2, y = b/2* and *z = h/2)*, in-plane shear stress components □ □ □ at *(x = 0, y = 0* and *z = h/2)*, transverse shear stress components □ □ □ at *(x = a/2, y = 0* and *z = 0)*, transverse shear stress components □ □ □ at *(x = 0, y = b/2* and *z = 0)* of simply supported rectangular plate (*a =1.5 b*) subjected to thermal load for aspect ratio 5.

Model	□	□	□	□ □	□ □	□ □ □	□ □ □	□ □ □
Present TSDT	0.7391	0.0309	0.0464	0.1122	0.0498	0.0748	-0.0156	-0.0235
HSDT	0.7391	0.0309	0.0464	0.1122	0.0498	0.0748	-0.0156	-0.0235
CPT	0.8686	0.0363	0.0545	0.0913	0.0180	0.0879	---	---

Table 2.2: Maximum transverse displacement □ at *(x = a/2* and *y = b/2)*, in-plane displacement components □ and □ at *(x = 0, y = b/2* and *z = h/2)* and *(x = a/2, y = 0* and *z = h/2)* respectively, normal stress components □ □ and □ □ at *(x = a/2, y = b/2* and *z = h/2)*, in-plane shear stress components □ □ □ at *(x = 0, y = 0* and *z = h/2)*, transverse shear stress components □ □ □ at *(x = a/2, y = 0* and *z = 0)*, transverse shear stress components □ □ □ at *(x = 0, y = b/2* and *z = 0)* of simply supported rectangular plate (*a = 1.5 b*)

ISSN: 2278-4632
Vol-10 Issue-3 No.01 March 2020

subjected to thermal load for aspect ratio 10.

Model	□	□	□	□□	□□	□□□	□□□	□□□
Present TSDT	1.4783	0.0154	0.0232	0.0561	0.0249	0.0374	-0.0039	-0.0058
HSDT	1.4783	0.0154	0.0232	0.0561	0.0249	0.0374	-0.0039	-0.0058
CPT	1.7373	0.0181	0.0272	0.0456	0.0090	0.0439	---	---

3. Conclusions:

The results are compared to those of other theories using an improved shear deformation theory for the thermal stress analysis of thick isotropic plates. A sample problem is taken into consideration to verify the effectiveness of the current Trigonometric shear deformation. The results for displacements and stresses are compared to the corresponding results of the higher order shear deformation theory (HOSDT) and the classical plate theory (CPT) (HSDT). Maximum non-dimensional axial and transverse displacements, as well as axial and shear stresses, are compared for

1. The results obtained by the present theory are accurate as seen from the comparison with exact results andare in general, superior to those of other refined shear deformation theories.
2. Transverse shear stresses obtained by integrating equilibrium equations (with respect to the thicknesscoordinate) satisfy shear stress free conditions on the top and bottom surfaces of the plate.
3. For simply supported plate subjected to thermal load, the transverse deflection given by present theory is inexcellent agreement with that of other higher order shear deformation theories.
4. The present theory gives realistic results of this displacement component in commensurate with the othershear deformation theories.

In general, the use of present theory gives accurate results as seen from the numerical example studied and it is capable of predicting the local effects in the simply supported plate. This validates the efficiency and credibility of trigonometric shear deformation theory.

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