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Planar graphs decomposable into a forest and a matching

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ABSTRACT

We showed that a planar graph of girth 11 can be decomposed into a forest and a matching. & improved the bound on girth to 9. We give sufficient conditions for a planar graph with 3-cycles to be decomposable into a forest and a matching.

***Keywords**:* Planar graphs; Edge decompositions; Graph decompositions

Introduction:-

*In computer science, a **graph** is an abstract data type that is meant to implement the undirected graph and directed graph concepts from the field of graph theory within mathematics.*

A graph data structure consists of a finite (and possibly mutable) set of vertices (also called nodes or points), together with a set of unordered pairs of these vertices for an undirected graph or a set of ordered pairs for a directed graph. These pairs are known as edges (also called links or lines), and for a directed graph are also known as edges but also sometimes arrows or arcs. The vertices may be part of the graph structure, or may be external entities represented by integer indices or references.

He et al. [3] proved that each planar graph with girth 11 or more can be decomposed into a forest and a matching (i.e., has an *FM-coloring*). Kleitman et al. [1] proved the same statement for planar graphs with girth at least 10. The restriction on girth was further improved to 9 by Borodin et al. [2]. Namely, the following was proved.

Theorem 1. *Every planar graph G of girth at least* 9 *has an FM-coloring.*

This implies (see [3]) that the game chromatic number and the game coloring number of every planar graph with girth at least 9 are at most 5.

The purpose of this note is to describe a broader class of sparse planar graphs whose edges can be decomposed into a forest and a matching. Due to the result in [2], we allow 3-cycles but impose some restrictions on the structureof these graphs.

By a *k-sunflower*, S_k, where $k \geqslant 4$, we mean the graph obtained from k-cycle C_k by putting a triangle on each edge so that one vertex of the triangle has degree 2 (there are k vertices of degree 2 and k vertices of degree 4).

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Claim 2. *No k-sunflower has an FM-coloring.*

Proof. Let $x_1, \ldots, x_k$ be the vertices of the basic cycle, C_k, in S_k (in this cyclic order), and for $i = 1, \ldots, k$, let y_i beadjacent to x_i and x_{i+1}. Suppose the edges of S_k are colored green and red so that the green edges form a forest, and the red edges, a matching. At least one edge, say x_1x_2, of C_k is red. Then x_2x_3 and x_2y_2 are green, and hence y_2x_3

should be red. Now, for the same reasons, x_3x_4 and x_3y_3 are green, and so y_3x_4 is red, and so on. Finally, we concludethat y_kx_1 is red, a contradiction to x_1x_2 being red. Q

Let G^* be obtained from a graph G by contracting triangles of G. Namely, at every step we identify the three vertices of a triangle of G into one vertex and delete the edges of this triangle, but every other edge of G survives. Inparticular, G^* may have parallel edges and loops. Note that by definition, no multigraph with a loop or a triple edge may have an *FM*-coloring.

Claim 3. *Suppose that G has no sunflowers, i.e. every ≥ 4-cycle in G has a nontriangular edge. If G^* has an FM- coloring then G also has an FM-coloring.*

Proof. Suppose that the sequence $G_0, \ldots, G_t$ of (multi)graphs is such that $G_0 = G$, $G_t = G^*$, and for $i = 0, \ldots, t-1$, graph G_{i+1} is obtained from G_i by contracting a triangle into a vertex. By assumption, G_t has an *FM*-coloring f_t. Suppose that for some $i \leq t-1$, G_{i+1} has an *FM*-coloring f_{i+1} and that G_{i+1} is obtained from G_i by contracting the triangle with vertices x, y, and z. For every edge $e \in E(G_i) - \{xy, yz, zx\}$, let $f_i(e) = f_{i+1}(e)$. Since f_{i+1} is an *FM*-coloring, at most one red edge is incident in G_i to the set $\{xy, yz, zx\}$. So, we may assume that no red edges are incident to y and z. In this case, color yz with red and xy and xz with green. Note that the existence of a green cycle in the new coloring f_i of G_i would imply that f_{i+1} also colors a cycle in G_{i+1} with green. Thus f_i is an *FM*-coloring of G_i. Repeating this step for $i = t-1, t-2, \ldots, 0$, we obtain an *FM*-coloring of $G_0 = G$. Q

Note the following structural property of G^*:

Claim 4. *Suppose every ≥ 4-cycle in a graph G has at least k nontriangular edges, where k ≥ 3; then the girth of G^* is at least k.*

Our main result is the following extension of Theorem 1.

Theorem 5. *If every ≥ 4-cycle in a planar graph G has at least 9 nontriangular edges, then G has an FM-coloring.*

It immediately follows from Claims 3 and 4 and Theorem 1 itself. As explained in [3], this implies:

Corollary 6. *The game chromatic number and game coloring number of every planar graph having no cycles with less than 9 nontriangular edges are at most 5.*

By $d_\Delta(G)$ denote the minimal distance between triangles in G.

Corollary 7. *Every planar graph G can be decomposed into a forest and a matching if at least* *one of the following holds:*

(i) $d_\Delta(G) \geq 1$ *and G has no cycles of length from* 4 *to* 16*;*
(ii) $d_\Delta(G) \geq 2$ *and G has no cycles of length from* 4 *to* 12*;*
(iii) $d_\Delta(G) \geq 4$ *and G has no cycles of length from* 4 *to* 10*;*
(iv) $d_\Delta(G) \geq 5$ *and G has no cycles of length from* 4 *to* 9*.*

To deduce (i) in Corollary 7 from Theorem 5, it suffices to note that if *G* has neither cycles of length from 4 to 16 nor two triangles with a common vertex, then *G* has no cycles with less than 9 nontriangular edges. (Note that the cubic graph obtained from the dodecahedron by cutting off all its corners has neither 3-cycles with a common vertex nor cycles of length 4 to 9, and it clearly has no FM-coloring. On the other hand, S_k has neither cycles of length from 4 to k − 1, nor FM-coloring.) Similarly, we deduce (ii)–(iv).

Accordingly, the graphs in Corollary 7 have the game chromatic number and game coloring number at most 5.

References

[1] A. Bassa, J. Burns, J. Campbell, A. Deshpande, J. Farley, M. Halsey, S. Michalakis, P.-O. Persson, P. Pylyavskyy, L. Rademacher, A. Riehl, M. Rios, J. Samuel, B. Tenner, A. Vijayasaraty, L. Zhao, D.J. Kleitman, Partitioning a planar graph of girth ten into a forest and a matching, (2004) (manuscript).

[2] O.V. Borodin, A.V. Kostochka, N.N. Sheikh, Gexin Yu, Decomposing a planar graph with girth nine into a forest and a matching, European J. Combin. (in press).

[3] W. He, X. Hou, K.W. Lih, J. Shao, W. Wang, X. Zhu, Edge-partitions of planar graphs and their game coloring numbers, J. Graph Theory 41 (2002) 307–317.

