# A RELATIVE DIAGNOSIS OF LAPLACE AND KAMAL TRANSFORMS 

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#### Abstract

There are numerous integral transforms which are being extensively usedto solve many of the real life, science and engineering problems. In this paper, two integral transforms namely Kamal transform and very famous Laplace transform are studied comparatively. Application of these transforms to solve linear difference equations is demonstrated. Study shows that these integral transforms are in close connection to each other.


Keywords: Laplace transform,Kamal transform, difference equation.

## 1. Introduction

The reality is that except change nothing is permanent in the natural world. Many processes and phenomena of the real world are described by the principles or laws that are expressed in the form of relations or statements involving rates of change. Mathematically,the rates are derivatives and the relations or statements are equations, and thus we have the differential equations. There are various type oftools and techniques among which Integral transform is one of the useful and effective tools for solving differential and integral equations.

Integral transforms are derived from the classical Fourier integral. An integral transform T of $f(r)$
, $\mathrm{r} 1 \leq \mathrm{r} \leq \mathrm{r} 2$ is defined in the following form:
$\int^{c} 2 \mathrm{~K}(\mathrm{~s}, \mathrm{r}) \underset{\mathrm{f}(\mathrm{r}) \mathrm{dr}, 1}{\mathrm{~T}[\mathrm{f}(\mathrm{r})]=}$
provided the integral exists.
The function $\mathrm{K}(\mathrm{s}, \mathrm{r})$ is called the kernel of a transform. There are numerous useful integral transforms [1-13] such as the Laplace, Fourier, Mellin, Hankel, Sumudu, Kamal, Aboodh, Mohand, etc., as classified by a choice of the kernel and the range of r. Many researchers used these transforms to solve the problems involving differential equations [1-4, 8, 14-15, 23, 36], integral equations [27, 35, 37], integro-differential equations [26], problems of growth and decay [24], etc. Aggarwal et al. [14-18] gave a comparative study of Mohand transform and Laplace, Kamal, Elzaki, Aboodh, Sumudu transforms that show the close connection between them. Aggarwal et al. [19-21] defined the dualities between Mohand transform and some useful integral transforms, Laplace transform and some useful integral transforms, Kamal transform and some useful integral transforms. Chaudhary et al. [22] described the connections between Aboodh transform and some integral transforms that include Laplace transform, Kamal transform, Elzaki transform, Sumudu transform, Mahgoub transform, Mohand transform and Sawi transform.

Abdelilah Kamal [8] proposedKamal transform in order to facilitate the process of solving ordinary and partial differential equations. Abdelilah and Hassan [8] solved ordinary differential equations by applying Kamal transform. Abdelilah et al. [23] solved partial differential equations by applying Kamal transform. Aggarwal and Chaudhary [15] solved the systems of ordinary differential equations by using Kamal transform. Aggarwal et al. [24-25, 27, 35, 37 ]solved linear Volterra integral equations of first and second
kind, population growth and decay problems, Abel's integral equation by using Kamal transform . Gupta et al. [26] solved Linear Partial Integro-Differential Equations by

Kamal Transform. Khan et al. [36] discussed the application of Kamal transform by solving ordinarylinear differential equations with variable coefficients.

In this paper, two integral transforms, namely, Kamal transform and very famous Laplace transform are studied comparatively by discussing some of their properties, transforms and inverse transforms of some frequently used functions, transform of derivatives and integrals. In the application part we solved one linear difference equation by using both Laplace and Kamal transforms.

## 2. Definition of Laplace and Kamal transforms

Laplace Transform

where Lis the Laplace transform operator.
Kamal Transform
In 2016,Abdelilah, K. and Hassan, S. [8], defined Kamal transform of the function $f(r), r \geq$

where K is the Kamal transform operator.
The two conditions for $\mathrm{f}(\mathrm{r}), \mathrm{r} \geq 0$, namely, Piecewise continuity and being of exponential order, are sufficient for existence of Laplace and Kamal transforms off(r).

## 3. Properties of Laplace and Kamal transforms Linearity property

(i) (Laplace transform) $[1-3,14]:$ If $\mathrm{L}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}_{\mathrm{L}}(\mathrm{s})$ and $\mathrm{L}\{\mathrm{g}(\mathrm{r})\}=\overline{\mathrm{g}}^{-} \mathrm{L}^{-}(\mathrm{s})$, then
$\mathrm{L}\{\alpha \mathbf{f}(\mathrm{r})+\beta \mathrm{g}(\mathrm{r})\}=\alpha \overline{\mathbf{f}} \mathrm{L}(\mathrm{s})+\beta^{-} \mathrm{g}^{-}(\mathrm{s}), \underline{\text { where }} \alpha, \beta$ are arbitrary constants.
(ii) (Kamal transform) $[15,24]$ :If $\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}$ (s) and $\mathrm{K}\{\mathrm{g}(\mathrm{r})\}=\overline{\mathrm{g}}^{--} \mathrm{K}$ (s), then
$\mathrm{K}\{\alpha \mathbf{f}(\mathrm{r})+\beta \mathrm{g}(\mathrm{r})\}=\alpha \overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{s})+\beta^{-} \mathrm{g}^{-} \mathrm{K}(\mathrm{s})$, where $\alpha, \beta$ are arbitrary constants.
First Shifting Property
(i) (Laplace transform) $[1-3,14]: I f L\{f(r)\} \equiv \overline{\mathbf{f}} \mathrm{L}(\mathrm{s})$, then $L\left\{\mathrm{e}^{\mathbf{c}} \mathbf{f}(\mathrm{r})\right\}=\overline{\mathbf{f}} \mathrm{L}(\mathrm{s}-\alpha)$. (ii) (Kamal transform) $[15]$ :If $\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\mathbf{f}^{--}$(s), then $\mathrm{K}\left\{\mathrm{e}^{\mathrm{c}}\{(\mathrm{r})\}=\overline{\mathbf{f}}^{-}\right.$

K Second Shifting Property
${ }^{(\mathrm{i})}$ wherge (Laplace transform)[1-2]:If $\mathrm{L}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}} \mathrm{L}(\mathrm{s})$ and $\mathrm{F}(\mathrm{r})=\mathbf{f}(\mathrm{r}-\alpha) \leq(\mathrm{r}-\alpha)$,

$=$ if $r>\alpha$, i.e., Fr
, then
$\mathrm{L}\{\mathrm{F}(\mathrm{r})\}=\mathrm{e}^{-\mathrm{c}} \overline{\mathbf{f}}_{\mathrm{L}}(\mathrm{s})$.
fr- $\alpha$, if $r>\alpha$
(ii) (Kamal transform):If $\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}$ (s), then $\mathrm{K}\{\mathrm{F}(\mathrm{r})\}=\mathrm{e}^{-/ \mathrm{c}} \overline{\mathbf{f}}^{-} \overline{\mathrm{K}^{-}}$(s). Proof: By definition,

$$
\begin{aligned}
& \mathrm{K}\{\mathrm{~F}(\mathrm{r})\}=\underset{0}{\mathbf{f}} \mathrm{e}^{-\overline{\mathrm{s}}} \stackrel{\mathrm{r}}{\mathrm{~F}(\mathrm{r}) \mathrm{dr}=\mathrm{f}^{-\overline{\mathrm{e}}^{\mathrm{r}} \mathrm{~s}} \mathrm{f}(\mathrm{r}-\alpha) \mathrm{dr}} \\
& =\int 0 \mathrm{e}^{-} \mathrm{s}^{-}(+\mathrm{u}) \mathrm{f}(\mathrm{n}) \mathrm{dn}(\text { putr }-\alpha=\mathrm{n})
\end{aligned}
$$

$-\frac{\mathbf{u}}{\mathrm{f}} \mathrm{f}(\mathrm{n}) \mathrm{dn}=\mathrm{e}$
$-/ \mathrm{c}_{\mathbf{f}} \overline{\mathrm{K}}^{-}(\mathrm{s})$.
$\underset{\text { (i) }}{\text { Chang of of scale Property }}$ transform) $[1-3,14]: \operatorname{If} L\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}(\mathrm{s})$, then $L\{\mathbf{f}(\alpha \mathrm{r})\}$
1 -_ $^{\mathrm{c}}$. $=\mathrm{f}_{\mathrm{L}}()$
(ii) (Kamal transform)[15]:If $\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{s})$, then $\mathrm{K}\{\mathbf{f}(\alpha r)\}$

1--- ( $\alpha$ s).

$$
=-f_{\mathrm{K}}
$$

## Convolution Theorem

(i) (Laplace transform) $[1-3,14]:$ If $\mathrm{L}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}} \mathrm{L}(\mathrm{s})$ and $\mathrm{L}\{\mathrm{g}(\mathrm{r})\}=\overline{\mathrm{g}}^{-} \mathrm{L}^{-}(\mathrm{s})$, then $\mathrm{L}\{\mathbf{f}(\mathrm{r}) * \mathrm{~g}(\mathrm{r})\}=\mathrm{L}\{\mathbf{f}(\mathrm{r})\} \mathrm{L}\{\mathrm{g}(\mathrm{r})\}=\overline{\mathbf{f}}_{\mathrm{L}}(\mathrm{s})^{-} \mathrm{g}_{\mathrm{L}}^{-}(\mathrm{s})$,
where $\mathrm{f}(\mathrm{r}) * \mathrm{~g}(\mathrm{r})={ }_{0}{ }^{\mathrm{c}} \mathrm{f}(\mu) \mathrm{g}(\mathrm{r}-\mu) \mathrm{d} \mu{ }_{0}=\int^{\mathrm{c}} \mathrm{f}(\mathrm{r}-\mu) \mathrm{g}(\mu) \mathrm{d} \mu$.
(ii) (Kamal transform) [15, 24-27]:If $\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}^{-}}$(s) and $\mathrm{K}\{\mathrm{g}(\mathrm{r})\}=\overline{\mathrm{g}}^{--} \mathrm{K}$ (s), then $\mathrm{K}\{\mathbf{f}(\mathrm{r}) * \mathrm{~g}(\mathrm{r})\}=\mathrm{K}\{\mathbf{f}(\mathrm{r})\} \mathrm{K}\{\mathrm{g}(\mathrm{r})\}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{s})^{-} \mathrm{g}^{-} \mathrm{K}(\mathrm{s})$,
where $\mathrm{f}(\mathrm{r}) * \mathrm{~g}(\mathrm{r})={ }_{0} \int^{\mathrm{c}} \mathrm{f}(\mu) \mathrm{g}(\mathrm{r}-\mu) \mathrm{d} \mu_{0}=\int^{c} \mathrm{f}(\mathrm{r}-\mu) \mathrm{g}(\mu) \mathrm{d} \mu$.
4. Laplace and Kamal transforms of some special functions

(ii) Kamal transformof $\mathrm{f}(\mathrm{r})$ is $\mathrm{K}\{\mathrm{f}(\mathrm{r})\} \underline{=1-\mathrm{el} \%} \mathrm{~s}^{\int_{Q^{2} / \mathrm{c}}} \mathrm{f}(\mathrm{r})$ dr

$$
1-\mathrm{e}-/ / \mathrm{s} \int_{0}
$$

Unit step functionor Heaviside function is defined as $\leq(r-\alpha)$
$= \begin{cases}0, & \text { if } r<\alpha \\ 1, & \text { if } r>\alpha\end{cases}$
(i) Laplace transform[1-3] of $\leq(r-\alpha)$ isL $\{\leq(r-\alpha)\}$

(ii) Kamal transform of $\leq(\mathbf{r}-\alpha)$ is $\mathrm{K}\left\{\leq(\mathrm{r}-\alpha) \dot{\}}=\mathrm{se}^{-/ \mathrm{c}}\right.$.

Proof: By definition,

| $\infty$ | $-r$ |
| :--- | :--- |
| $\infty$ | $-r$ |

$\mathrm{K} \overline{\{ } \leq(\mathrm{r}-\alpha)\} \stackrel{-}{=} \int_{0}$ e $\mathrm{s} \leq(\mathrm{r}-\alpha) \mathrm{dr}=\int$ e s dr $=\mathrm{se}$
Dirac delta functionor unit impulse function is defined as
(i) Laplace transform [1] of $\partial(\mathrm{t}-\alpha)$ is $\mathrm{L}\{\partial(\mathrm{t}-\alpha)\}=\mathrm{e}^{-\mathrm{c}}$.
(ii) Kamal transform of $\partial(t-\alpha)$ is $K\{\partial(t-\alpha)\}=e^{-/ c}$.

Proof: By definition,

$$
\mathrm{K}\{\partial(\mathrm{t}-\alpha)\}=\mathrm{f} \mathrm{e}
$$


$\mathrm{s}_{1} \mathrm{dt}$

$$
=\mathrm{se}^{-} / \mathrm{c} \lim _{\rightarrow \rightarrow 0+} \frac{1-\mathrm{e}-1 / \mathrm{s}}{\mathrm{~s}} \underset{\mathrm{c}}{\mathrm{~s}} \mathrm{e}^{-} / \mathrm{c} 1=\mathrm{e}^{-/ \mathrm{c}}
$$

5. Laplace and Kamal transforms of the derivatives

Laplace transforms [1-3, 14]:
If $\mathrm{L}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}} \mathrm{L}(\mathrm{s})$, then
(i) $\mathrm{L}\left\{\mathbf{f}^{\prime}(\mathrm{r})\right\}=\mathrm{s} \overline{\mathbf{f}}_{\mathrm{L}}(\mathrm{s})-\mathbf{f}(0)$
(ii) $\mathrm{L}\left\{\mathrm{f}^{\prime \prime}(\mathrm{r})\right\}=\mathrm{s}^{2} \overline{\mathbf{f}} \mathrm{~L}(\mathrm{~s})-\mathrm{s} \mathbf{f}(0)-\mathbf{f}^{\prime}(0)$
(iii) $\left.\mathrm{L} 1 \mathbf{f}^{(>)}(\mathrm{r})\right\}=\mathrm{s}^{>} \hat{\mathbf{f}}_{\mathrm{L}}(\mathrm{s})-\mathrm{s}^{>-\mathbf{1}} \mathbf{f}(0)-\mathrm{s}^{>-2} \mathbf{f}^{\prime}(0)-\ldots-\mathbf{f}^{(>-1)}(0)$

Kamal transforms [8, 15, 24]:
If $K\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}_{\mathbf{1}}^{-} \overline{\mathrm{K}}$ (s), then
(i) $\mathrm{K}\left\{\mathrm{f}^{\prime}(\mathrm{r})\right\}=\frac{1}{\mathrm{~K}} \mathrm{f}(\mathrm{s})-\mathrm{f}(0)$
(ii) $K\left\{\mathbf{f}^{\prime \prime}(\mathrm{r})\right\}^{\mathrm{c}^{c}} \overline{\mathrm{c}}^{2}{ }_{1} \overline{\mathbf{k}}_{c}^{--}(\mathrm{s})-^{1} \quad \mathbf{f}(0)-\mathbf{f}^{\prime}(0)-$
(iii) $K 1 \mathbf{f}^{(>)}\left(\int^{2} \frac{1}{=} \mathbf{f}^{c-}\right.$ (s)-


$$
\overline{\overline{\mathrm{cm}}-1} \underset{\mathrm{~cm}-2}{ }
$$

6. Laplace and Kamal transforms of integrals

Laplace transforms [1-3, 14]:
If $\underline{L}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}$ (s), then $\mathrm{L1}{ }^{\mathrm{c}} \quad(\mathrm{)}$
$\left.-\quad \int_{0} f \mu \mathrm{~d} \mu\right\}=\mathrm{c}_{\mathrm{L}}(\mathrm{s})$

Proof: By definition,
$\{\mathrm{f} f(\mu) \mathrm{d} \mu\} \mathrm{dr}$

$$
\mathrm{K}\left\{\underset{\mathbf{F}}{\mathbf{f}(\mu) \mathrm{d} \mu\}}{ }^{\infty}=\underset{\mathbf{F}}{ } \mathrm{e}^{-\mathrm{c} / \mathrm{c}} \mathrm{c}\right.
$$

$$
\begin{array}{lll}
0 & 0 & 0
\end{array}
$$

Changeof order of integration gives, $\infty$

$$
\mathrm{K}\{\stackrel{\mathrm{c}}{\mathbf{f}} \mathbf{f}(\mu) \mathrm{d} \mu\}=\boldsymbol{f} \mathbf{f}(\mu)
$$

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$$
\begin{aligned}
& \begin{array}{lll}
0 & 0 & \mu \\
\infty & \infty
\end{array} \\
& \left.=\mathbf{f} f(\mu) 1 \mathrm{se}^{-\mu / \mathrm{c}}\right\} \mathrm{d} \mu=\mathrm{s} \mathbf{f} \mathrm{e}^{-\mu / \mathrm{c}} \mathrm{f}(\mu) \mathrm{d} \mu \\
& 00 \\
& =\mathrm{s} \overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{~s})
\end{aligned}
$$

7. Laplace and Kamal transforms of $\mathrm{rf}(\mathrm{r})$

Kamal transforms ofrf(r):
$\stackrel{*}{\text { If }} \stackrel{\mathrm{c}}{\mathrm{K}}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}^{--}(\mathrm{s})$, then $\left.\mathrm{K}\{\mathrm{r} \mathbf{f} \underline{\mathrm{r}})\right\}=\mathrm{s}^{*} \quad \overline{\mathbf{f}}^{--}(\mathrm{s})$.
$\underset{*}{\text { Proof: }} \mathrm{K}^{\text {By definition, }}$

$$
\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\mathbf{f}^{\infty} \mathrm{e}^{-\mathrm{c} / \mathrm{c}} \mathbf{f}(\mathrm{r}) \mathrm{dr}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{~s})
$$

On differentiating w.r.t.s and using Leibnitz's rule for differentiation under integral sign, we get
W _r



* c K

8. Duality between Laplace and Kamal transforms [20-21]

If $\mathrm{L}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}_{\mathrm{L}}(\mathrm{s})$ and $\mathrm{K}\{\mathbf{f}(\mathrm{r})\}=\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{s})$, then
$\overline{\mathbf{f}} \mathrm{L}(\mathrm{s})=\overline{\mathbf{f}}^{-} \overline{\mathbf{K}}(1 / \mathrm{s})$ and $\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{s})=\overline{\mathbf{f}}_{\mathrm{L}}(1 / \mathrm{s})$.
All the results stated above (in section 3 to 7) about Kamal transform can be obtained from
corresponding results about Laplace transform by using above mentioned duality between thesetransforms.
With the help of above mentioned duality between Laplace and Kamal transforms, we are giving the Kamal transforms of mostly used functions [15, 19-21] as listed in the Table1.


| 4 | $\mathrm{r}^{\mathrm{n}}, \mathrm{n} \in \rightarrow$ | n ! $\mathrm{s}^{\mathrm{n}+1}$ |
| :---: | :---: | :---: |
| 5 | $\mathrm{r}^{\mathrm{n}}, \mathrm{n}>-1$ | $\Gamma(\mathrm{n}+1)$ |
|  | ac |  |

$\mathrm{s}^{\mathrm{n}+1}$
$\qquad$ $s-a \sin a r$
$s-a \sin a r$
cos ar
-

$$
\begin{gathered}
\mathrm{a} \\
\mathrm{~s}^{2}+\mathrm{a}^{2} \\
\mathrm{~s}^{2}+\underset{\mathrm{a}}{ }+\mathrm{a}^{2}
\end{gathered}
$$

$\qquad$ 10
$s^{2}-\mathrm{a}^{2} \sinh \mathrm{ar}^{\sin }$
cosh ar
L

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$1-\underset{\sqrt{ } 1+\mathrm{s}^{2}}{ }$.

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$$
1-\sqrt{ } 1+\mathrm{s}^{2} \quad \begin{aligned}
& \sqrt{ } 1+\mathrm{s}^{2} \\
& \hline
\end{aligned}
$$

$$
\operatorname{erf}(\sqrt{ } x)=
$$

$\underset{\sqrt{n} 0}{2 \underset{f}{f x}} \mathrm{e}^{-c^{2}} \mathrm{dr}$
$=$

9. Inverse Laplace and Kamal transforms
${ }_{(i)}$ Inverse Laplace transforms [1-3, 14]: If $\overline{\mathbf{f}} \mathrm{L}(\mathrm{s})$ is the Laplace transform of $\mathrm{f}(\mathrm{r})$, then $\mathbf{f}(\mathrm{r})$ is called the inverse Laplace transform of $\overline{\mathbf{f}} \mathrm{L}(\mathrm{s})$ and it is expressed as $\mathbf{f}(\mathrm{r})=\mathrm{L}^{-1}\{\overline{\mathbf{f}} \mathrm{~L}(\mathrm{~s})\}$,
$\mathrm{L}^{-1}$ is an inverse Laplace transform operator.
${ }_{\text {(ii) }}$ Inverse Kamal transforms [15, 24-25]: If $\overline{\mathbf{f}}^{-} \overline{\mathbf{K}^{-}}(\mathrm{s})$ is the Kamal transform of $\mathrm{f}(\mathrm{r})$, then $\mathbf{f}(\mathrm{r})$ is called the inverse Kamal transform of $\overline{\mathbf{f}}^{-} \overline{\mathrm{K}^{\prime}}$ (s) and it is expressed as $\mathbf{f}(\mathrm{r})=\mathrm{K}^{-1}\left\{\overline{\mathbf{f}}^{-} \overline{\mathrm{K}}(\mathrm{s})\right\}$,
$\mathrm{K}^{-1}$ is an inverse Kamal transform operator.
Inverse Laplace transforms [1-3, 14] and inverse Kamal transforms [15] of some frequently usedfunctions are given below in Table- 2.

| Table2. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| S.N. | $\overline{\mathbf{f}}(\mathrm{s})$ | $\mathrm{L}^{-1}\{\overline{\mathbf{f}}(\mathrm{~s})\}=\mathrm{K}^{-1}\{\overline{\mathrm{~g}}(\mathrm{~s})\}$ |  | $\overline{\mathrm{g}}$ (s) |
| 1 | 1 |  |  |  |
|  | - |  | s |  |
| 2 | 1 |  |  |  |
|  | - |  | $\mathrm{s}^{2}$ |  |
| 3 | 1 |  |  |  |
| 4 | - |  | $\mathrm{r}^{\text {n }} \quad \mathrm{s}^{3}$ |  |
|  | s |  |  |  |
|  |  |  | r | $\mathrm{s}^{2}$ |
|  |  |  | $\mathrm{r}^{2} / 2$ ! | $\mathrm{s}^{3}$ |
|  |  |  | $\mathrm{s}^{\mathrm{n}+1}$ |  |


$7 \quad \overline{1} \quad \mathrm{~s}-\mathrm{a}$
$1 \mathrm{e}^{\mathrm{ac}} \quad \frac{\mathrm{s}}{\Gamma_{\mathrm{s}^{2}}^{\text {as }}}$
$\frac{}{1+\mathrm{a}^{2} \mathrm{~s}^{2}}$

$$
s^{2}+a^{2}
$$

$$
1-a^{2} s^{2}
$$

$10 \quad \mathrm{~s}$

$$
\underset{\mathrm{s}^{2}}{\mathrm{~s}}+\mathrm{a}^{2} \mathrm{~s}^{2}
$$

1

| 8 | s |
| :--- | :--- |
|  | 1 |


| cos ar | s |
| :---: | :---: |
| 1 | $1+\mathrm{a}^{2} \mathrm{~s}^{2}$ |

$10-\frac{a^{\sinh \mathrm{ar}}}{10}$

$$
\mathrm{s}^{2}-\mathrm{a}^{2}
$$

$$
1 \stackrel{\mathrm{~s}}{\mathrm{a}^{2} \mathrm{~s}^{2}}
$$

cosh ar

$$
\frac{1}{\sqrt{1}^{1+s^{2}}}
$$

12

$$
1-\underline{s}
$$

$$
\sqrt{ } 1+s^{2}
$$

$$
\mathrm{J}_{0}(\mathrm{r}) \text {, a Bessel's function of orderzero }
$$

$\mathrm{J}_{1}(\mathrm{t})$, a Bessel's function of orderone
Error function
$\square$

$$
\begin{aligned}
& \sqrt{ } 1+\mathrm{s}^{2} \\
& 1-{ }_{1}-\mathrm{s}^{2}
\end{aligned}
$$

$$
\mathrm{s} \sqrt{ } \mathrm{~s}+1
$$


10. Application of Laplace and Kamal transforms for solving linear difference equations Many researchers applied these transforms to solveordinary and partial differential equations [8, 23], ordinary linear differential equations with variable coefficients [36], system of ordinary differential equations [14-15], linear Volterra integral equations of first kind [27] and second kind [35], population growth and decay problems [24], Abel's integral equation [37], linear partial integro-differential equations [26]. In this section we solve the linear difference equation using Laplace and Kamal transforms.

Linear difference equations: A linear difference equation is a linear relation between functions $\mathrm{y}(\mathrm{r}), \mathrm{y}(\mathrm{r}+h), \mathrm{y}(\mathrm{r}+2 h), \ldots, \mathrm{y}(\mathrm{r}+\mathrm{n} h)$, where $h$ (a constant) is the given interval of differencing andn is a given integer.

Example: Solve

$$
\begin{equation*}
4 y(r)-5 y(r-1)+y(r-2)=r^{2} \text { for all } r>0 \tag{10.1}
\end{equation*}
$$

given that $\mathrm{y}(\mathrm{r})=\lambda$ whenr $\leq 0$.
Solution using Laplace transforms [34]:
Taking the Laplace transform of eq. (10.1), we have

$$
\begin{equation*}
4 \mathrm{~L}\{\mathrm{y}(\mathrm{r})\}-5 \mathrm{~L}\{\mathrm{y}(\mathrm{r}-1)\}+\mathrm{L}\{\mathrm{y}(\mathrm{r}-2)\}=\mathrm{L}\left\{\mathrm{r}^{2}\right\} \tag{10.2}
\end{equation*}
$$

i.e., $\quad 4 \overline{\mathrm{y}} \mathrm{L}-5 \mathrm{~L}\{\mathrm{y}(\mathrm{r}-1)\}+\mathrm{L}\{\mathrm{y}(\mathrm{r}-2)\}=2 / \mathrm{s}^{3}$
where $\mathrm{L}\{\mathrm{y}(\mathrm{r})\}=\overline{\mathrm{y}} \mathrm{L}=\overline{\mathrm{y}} \mathrm{L}(\mathrm{s})$.
Now, $L\{y(r-n)\}=\int^{\infty} e^{-c c} y(r-n) d r=\int^{\infty} e^{-c(x+n)} y(x) d x(x=r-n)$
$=e^{-c n}{ }_{0}$
$\int^{\infty} e^{-c x} y(x) d x+e^{-c}$

$$
\int_{-\mathrm{n}}^{0}
$$

$\mathrm{e}^{-\mathrm{c}}$
$\lambda d x \quad(y(x)=\lambda$, when $r \leq 0)$
Hence (10.2) becomes
$=\mathrm{e}^{-\mathrm{cn}} \bar{y}_{\mathrm{L}}$
$+{ }^{\lambda}\left(1-e^{-c n}\right) s$
$\stackrel{+}{\left.{ }_{(1-2} \sum^{\infty}-2 n-2\right)^{e}} \mathrm{e}^{-\mathrm{n}}$

$$
\begin{aligned}
& \bar{y}=;+1 \\
& L^{(10.3)} 2 \mathrm{c}^{3}
\end{aligned}
$$

$3 \mathrm{n}=1$ c3
Taking inverse Laplace transform of eq. (10.3), we have

$$
\mathrm{y}(\mathrm{r})=\lambda+{ }^{1} \underline{\mathrm{r}}^{2}+{ }_{{ }_{2}}^{\infty} \sum^{\infty}\left(1-2^{-2 \mathrm{n}-2}\right) \leq(\mathrm{r}-\mathrm{n})(\mathrm{r}-\mathrm{n})^{2}
$$

Since (

$$
\left.\begin{array}{rl} 
& 3_{n=1}^{0,} \text { if } r \leq n \\
\leq r-n
\end{array}\right\}\left\{\begin{array}{l}
\quad \leq \quad \text { if } r>n
\end{array}\right.
$$

, the solution becomes
where $[\mathrm{r}]$ is the greatest integer not exceeding r .

## Solution using Kamal transforms:

Taking the Kamal transform of eq. (10.1), we have

$$
4 \mathrm{~K}\{\mathrm{y}(\mathrm{r})\}-5 \mathrm{~K}\{\mathrm{y}(\mathrm{r}-1)\}+\mathrm{K}\{\mathrm{y}(\mathrm{r}-2)\}=\mathrm{K}\left\{\mathrm{r}^{2}\right\}
$$

$$
\begin{equation*}
\text { i.e., } \quad 4 \bar{y}_{K}-5 \mathrm{~L}\{\mathrm{y}(\mathrm{r}-1)\}+\mathrm{L}\{\mathrm{y}(\mathrm{r}-2)\}=2 \mathrm{~s}^{3} \tag{10.5}
\end{equation*}
$$

where $\mathrm{K}\{\mathrm{y}(\mathrm{r})\}=\overline{\mathrm{y}} \mathrm{K}=\overline{\mathrm{y}} \mathrm{K}(\mathrm{s})$.
Now, by definition $K\{y(r-n)\}=\int^{\infty} e^{-c / c} y(r-n) d r$.
Following the procedure as described in case of Laplace transform or using the duality betweenLaplace and Kamal transform, we have

$$
\mathrm{K}\{\mathrm{y}(\mathrm{r}-\mathrm{n})\}=\mathrm{e}^{-\mathrm{n} / \mathrm{c}} \overline{\mathrm{y}} \mathrm{~K}+\mathrm{ks}\left(1-\mathrm{e}^{-\mathrm{n} / \mathrm{c}}\right)
$$

Hence (10.5) becomes


Taking inverse Kamal transform of eq. (10.6) and using

which is the same as obtained by Laplace transforms.

## 11. Results and Discussions

In this paper, we made a comparative discussion on Laplace and Kamal transforms through some of their properties, transforms and inverse transforms of some frequently used functions, derivatives and integrals. Further, in the application section we solved a linear difference equation using both the transforms. Study shows that both the transform techniques are parallel and closely connected to each other by the duality relation mentioned in section (8).

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