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A STUDY ON MAXIMA AND MINIMA FORSINGLE REAL VALUED FUNCTION

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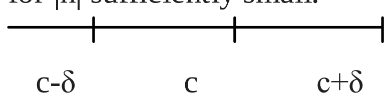
ABSTRACT

In calculus, when we refer to a function's maximum value, we do not necessarily mean the function's utmost possible value. When f (c) exceeds all possible values that f (x) might adopt in the vicinity of x = c, f (x) is said to be maximal for that value of x. In a similar manner, a minimal value of f (x) is defined as the value that is lower than values in the immediate vicinity.

Keywords: Function, derivatives, turning value, critical values, logical reasoning, substantial are some of the most important terms.

INTRODUCTION AND MOTIVATION

(i) Maximum value of a function: A function f (x) is said to have a maximum value for x = c provided we can get a positive quantity δ such that for all values of x in the interval $c-\delta < x < c+\delta$ $(x \neq c)$ $f(c) > f(x)$ i.e., if $f(c+h) - f(c) < 0$ for $|h|$ sufficiently small.



Minimum value of a function: The function f (x) has a minimum value for x = c provided we can get an interval $c - \delta' < x < c + \delta'$ within which $f(c) < f(x)$ $(x \neq c)$. i.e., if $f(c+h) - f(c) > 0$ for sufficiently small. The maximum and minimum values of a function are also known as relatively greatestand least values of the function in that these are the greatest and the least

(ii) values of the function relatively to some neighbourhoods of the points in question. The extreme value is used both for a maximum as well as for a minimum value. And while ascertaining whether a value f (c) is an extreme value of f or not, we compare f (c) with the values of f for values of x in any neighbourhood of c. So that the values of the function outside the neighbourhood do not come into question. Thus a maximum (minimum) values of a function may not be the greatest (least) of all the values of the function in a finite interval. In fact a function can have

several maximum and minimum values and a minimum value may even be greatest than a maximum value (Das, & Mukharje, 1986).

OBJECTIVE

- Through the study of this article, we should understand the master the conditionsandconclusionofmaximumandminimumvalueofthefunction f (x). We should understand the necessary condition for maximum and minimum and the geometrical interpretation of maximum and minimum.
- Through the study of this article we should know the some of the theorem about of it. We also know how to use and it improves logical thinking ability of providing mathematical proposition.

METHODOLOGY

We can determine maxima and minima of f (x) by proceeding the working rule as equate f' (x) to zero and let the roots be c_1, c_2, c_3 to work out the value of f'' (c_1), if it is negative, then x = c_1 makes f (x) is maximum. if f'' (c_1) be positive, then f (c_1) is a minimum of f (x). Similarly test the sign of f'' (x) for the other values c_2, c_3. of x for which f' (n) is zero and determine whether f (x) is a maximum or a minimum of these points. The described expression for determining maxima and minima of f (x) fails at the paint where f' (x) is non existence even through f (x) may be continuous there (Narayan, 1988). In such a case we should bear in mind that if (x) be maximum at a point, immediately to the left of it the value of f (x) is less, and gradually increases towards the value at the point and so f' (x) is

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positive. Immediately to the right the value of f (x) is again less and so f (x) decreases with x increasing and therefore f' (x) is negative to the right. Thus f' (x) changes sign from positive on the left to negative towards the right of the paint. Similarly, if f (x) be a minimum at any paint f (x) is larger on the left and diminishes to the value at the point and again becomes larger on the right i.e., f (x) increases to the right, thus f' (x) changes sign here being negative on the left and positive on the right of the point (Coddington, 1998).

(A necessary condition for maximum and minimum.)

Theorem: If f(x) be a maximum or minimum at x = c and if f'(c) exists, then f'(c) = 0

Proof: We know by the definition, f (x) is maximum at x = c we can find a positive number δ such that $f(c + h) - f(c) < 0$ whenever $-\delta < h < \delta$ $(h \neq 0)$

$$\frac{f(c+h) - f(c)}{h} < 0$$ if h be positive and sufficiently small and > 0 if h be negative and numerically small.

Thus

Lt h '' 0

$$\frac{f(c+h) - f(c)}{h} \leq 0$$ and similarly

Lt h '' 0

$$\frac{f(c+h) - f(c)}{h} \geq 0.$$

Now, if f' (c) exists the above two limits which shows that right hand and left-hand derivatives of f (x) at c = 0 must equal respectively. Hence the only common value of the limit is zero. Thus f' (c) = 0.

As the same way exact similar is the proof when f (c) is minimum.

Determination of maxima and minima

Theorem: If c be a point in the interval in which the function f (x) is defined and if f'(c) = 0 and f'' (0) $\neq$ 0 then f (c) is

(i) a maximum if f'' (c') is negative and (ii) a minimum if f'' (c) is positive.

Proof: Suppose f' (c) = 0 and f'' (c) exists and $\neq 0$.

By the mean value theorem.

$$f(c+h) = hf'(c+\theta h),\ 0 < \theta < 1.$$

$$= ih^2 \cdot \frac{f(c+ih) - f(c)}{ih}$$

Since $0 < \theta < 1$, $\theta h \to 0$ as $h \to 0$ and writing $\theta h = k$ the coefficient

of θh^2 on the right side

$$\text{Lt}_{h \to 0} \frac{f'(c+k) - f'(c)}{k}$$

= f'' (c). According since

θh^2 is positive, f (c+h) - f (c) has the same sign as that of f'' (c) when |h| sufficiently small.

If f'' (c) is positive f (c+h) - f (c) is positive whatever h is positive or negative provided |h| is small. Hence f (c) is minimum.

Similarly, if f'' (c) is negative f (c+h) - f (c) is negative whether h is positive or negative, when |h| is mall and so f (c) is a maximum (Singh, & Bajracharya, 1998).

(ii) **Theorem:** let c be an interior point of the interval of definition of the function f (x), and let,

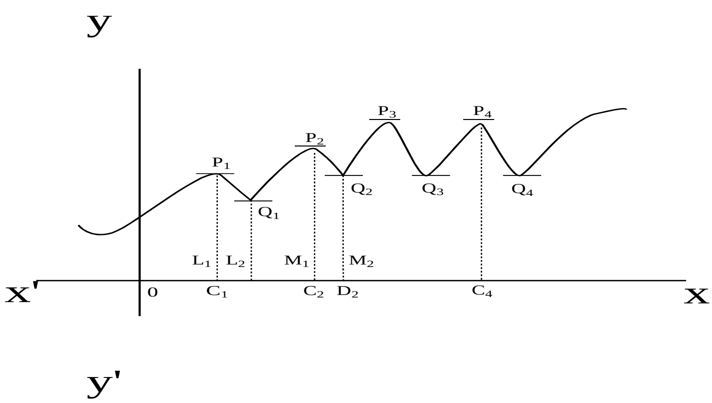
$$f'(c) = f''(c) - \ldots\ldots\ldots = f^{n-1}(c) = 0 \text{ and } f^n(c) \neq 0.$$

Then (1) if n is even, f (c) is a maximum or a minimum according as f^n (c) is negative or positive and (iii) if n be odd, f (c) is neither a minimum nor maximum.

(c+h) - f (c) has the save sign as of f^n (c), whether h is positiveor negative, provided |h| is sufficiently small. Hence, if f^h (c) be positive f (c+h) - f (c) is positive for either sign of h when |h| is small and so f (c) is aminimum.

Similarly if f^n (c) is negative f (c) is a maximum. Now suppose n is odd, then $\theta h^n/(n-1)!$ is positive or negative according as h is positiveor negative. Hence f (c+h) - f (c) changes in sign with the change of h whatever the sign of f^n (c) may be and so f (c) cannot be either a maximumor a minimum at x = 0 (Gupta, & Malik, 2000).

Graphical Interpretation: The following figure which represents graphically the f (x). A glance at the adjoining graph shows that the function has a maximum value at P_1 as also at P_2, P_3, P_4 etc. and has minimum values at Q_1, Q_2, Q_3, Q_4 etc. At P_1 for instance, corresponding to x = OC_1 (c_1 say), the value of the function namely, the ordinate P_1C_1 is not necessarily biggerthan the value Q_2D_2 at x = OD_2 but we can get a range say $L_1C_1L_2$ in the neighbourhood of C_1 on either side of it. (i.e., we can find a $\delta = L_1C_1 = C_1L_2$) such that for every value of x within $L_1C_1L_2$. The value of the function is less than P_1C_2. Hence by the definition, the function is maximum at x = OC_1.



Similarly, we can find out an interval $M_1D_2M_2$ ($M_1D_2 = D_2M_2 = \delta'$) in the neighbourhood of D_2 within which for every other value of x the function is greater than that at D_2. Hence the function at D_2 (Q_2D_2) (Goyal, & Gupta, 1999) is a minimum. From the figure we have regarding the factsmaxima and minima of a continuous function will be as conclude.

- that the function may have several maxima and minima in the interval.

- that maxima value of the function at some point may be less that a minimum value of it another point ($C_1P_1 < D_2Q_2$).
- Maximum and minimum values of the function occur alternatively i.e. between any two consecutive maximum values there is a minimum value and vice-versa.

FINDINGS

- To sum up, we arrived at the following conclusion . At last we conclude that for the maxima and minima. At a point where f (x) is maxima or a minima f' (x) changes sign from positive on the left to negative on the right, if f (x) be a maximum and from negative on the left to the positive on the right if f (x) be a minimum.
- If f' (x) exists at such points, it changes the sign from one side to another side take place through the zero value of f' (x), so that f' (x) = 0 at the point. if f' (x) be non-existent at the point the left-hand and right hand derivatives are of opposite signs of the point
- Even in the case where the successive derivatives exist, instead of proceeding to calculate at a paint to apply the usual criteria for maxima and minima of f (x) at the point, we may apply effectively in many cases, the simple criterion of changing of sign of f' (x+h) as h is changed from negative to positive value being numerically small.
- At point where f (x) is a maximum or a minimum f' (x) = 0 when it exists and accordingly at these points the tangent lines to the graph off (x) will be parallel to the x-axis (as at points P_1, Q_1, P_2, Q_2, P_3, Q_3, etc as in the above graph (fig). At points where f (x) is a maximum or a minimum, but f' (x) does not exists; the tangent line to the curve changes its direction abruptly while passing through the point. A special case where the tangent is parallel to the y-axis the change in the sign off' (x) taking place through an infinite value.
- A maximum or minimum is often called an "extremum" (external) or 'turning value' the value of x for which f' (x) or 1/f' (x) = 0 are often called "critical values" or critical point of f (x).

CONCLUSION

This paper through on the single variable function in the case of

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maxima and minima in teaching content optimizing the concept of use and improves the logical thinking of proving mathematical proposition and it is in connection with the subject related to teaching of mathematics. We consider that, the subject is not new, it is possible that some of our results exists in some forms in the literature. In this case we consider that there is something new idea in this approach. We hope to investigate further the subject in connection to the viewed as a starting point for driving more substantial results on the subject.

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