

Statistical analysis and scientific inquiry

1.Mr.Arijit Ghosh .2.Ms. Swatika Mohanty
Nalanda Institute of Technology, Bhubaneswar
Dept. of Basic Science & Humanities
E-mail ID: arijitgosh@thenalanda.com
swatikamohanty@thenalanda.com

Abstract.

A brief mention of probability's involvement in scientific inquiry is made. The traditional theory of probability and the relative theory are given particular prominence. The probabilities of single events, combined occurrences, and conditional events are described. Normal and binomial distribution properties are described. The central limit theorem, systematic and random mistakes, and sampling theory are all discussed..

Key words:

Distribution, estimation, sampling, probability

Introduction

Probability theory plays an important role in statistics and therefore, in scientific research. In this paper the authors explore three areas of probability; simple events, combinations of events, and the binomial distribution (Foroughbakhch & Badii, 2005; Mehta & Hilton, 1993; Mosteller, et al., 1961; Parzen, 1960).

The *classic theory* of probability underlines much probability in statistics. Briefly, it states that the chance of a particular outcome occurring is determined by the ratio of the number of favorable outcomes (or “successes”) to the total number of the outcomes. Expressed as a formula, $P(A) = \text{number of favorable outcomes} / \text{total \# of possible outcomes}$.

For example, the probability of randomly drawing an ace from a well-shuffled deck of cards is equal to the ratio $4/52$. Four is the number of favorable outcomes (the number of aces in the deck), and 52 is the number of total outcomes (the number of cards in the deck). The probability of randomly selecting an ace in one draw from a deck of card is therefore $4/52$, or 0.077. In statistical analysis, probability is usually expressed as a decimal and ranges from a low of 0 (no chances) to a high of 1.0 (certainty).

The classic theory assumes that all outcomes have equal likelihood of occurring. In the example just cited, each card must have an equal chance of being chosen-no card is larger than any other or in any way more likely to be chosen than any other card.

The classic theory pertains only to outcomes that are mutually exclusive (or disjoint), which means that those outcomes may not occur at the same time. For example, one coin flip can result in a head or a tail, but one coin flip cannot result in a head and a tail. So the outcome

of a head in the outcome of a tail are said to be “mutually exclusive” in one coin flip, as is the outcome of an ace and a king as the outcome of one card is being drawn.

Relative frequency theory

This theory holds that if an experiment is repeated an extremely large number of times, and a particular outcome occurs a percentage of the time, then this particular percentage is close to the probability of that outcome.

For example, if a machine produces 10,000 items on at a time, and 1,000 of those items are faulty, the probability of that machine producing a faulty item is approximately 1,000 out of 10,000, or 0.10.

Probability of simple events

What is the probability of simultaneously flipping three coins—a small, a mid-sized, and a large-sized, and having all three land heads?

Using the classic theory, determine the ratio of number of favorable outcomes to the number of total outcomes. Listed below are all possible outcomes (Table 1).

Table 1. Possible outcomes of eight trials.

<u>Outcome #</u>	<u>Small</u>	<u>Medium</u>	<u>Large</u>
1	H	H	H
2	H	H	T
3	H	T	H
4	H	T	T
5	T	H	H
6	T	H	T
7	T	T	H
8	T	T	T

There are 8 different outcomes, only one of which favorable (outcomes #1: all three coins landing heads); therefore, the probability of three coins landing heads is $1/8$, or 0.125.

What is the probability of exactly two of the three coins landing heads? Again, there are the 8 total outcomes, but in this case only three favorable outcomes (outcomes numbers 2, 3 and 5); thus, the probability of exactly 2 of 3 coins landing heads is $3/8$ or 0.375.

Independent Events

Each of the three coins being flipped about is what is known as an independent event. Independent events are defined as outcomes that are not affected by others outcomes. In other words, the flip of the small coin does not affect the flip of the mid-sized coin, and vice versa.

Dependent Events

Dependent events, on the other hand, are outcomes that are affected by others outcomes. For example, what is the probability of randomly drawing an ace from a deck of cards and then drawing an ace again form the same deck of cards, without returning the first drawing card back to deck?

For the first draw, the probability of a favorable outcome is $4/52$, as explained before; however, once that the first card has been drawing, the total number of outcomes is no longer 52, but now 51 because a card has been removed form the deck. And if the first card drawn resulted in a favorable outcome (an ace), there would now be only three aces in the deck. Or the number of favorable outcome would remain a four if the first card drawn were not an ace. So the second draw is a dependent event because its probability chance depending upon what happens on the first draw.

If, however, you replace that drawn card back into the deck and shuffle well again before the second draw, then the probability for a favorable outcomes for each draw will now be equal ($4/52$) and this events will be independent (Foroughbakhch & Badii, 2005; Hodges & Lehman, 1964; Hoel et al., 1972; Kemeny et al., 1958).

Probability of joint occurrences

Another way to compute the probability of all three flipped coins landing heads is as series of three different events: First flip small one, then flip the mid-sized one, then flip the largest one. Will the probability of landing three heads still be 0.25?

Multiplication rule

To compute the probability of two or more independent events, all occurring- joint occurrence- multiply their probabilities. For example, the probability of the small coin landing heads is $\frac{1}{2}$, or 0.5; the probability of the mid-sized coin next landing heads is $\frac{1}{2}$, or 0.5; and the probability of the large coin landings heads is $\frac{1}{2}$, or 0.5; those, note that. $0.5 \times 0.5 \times 0.5 = 0.125$, which is what you determine with the classic theory by assessing the ratio of number of favorable outcomes to number of the total outcomes. The notation for joint occurrence is $P(AB)=P(A) \times P(B)$ and reads: The probability of A and B both happening is equal to the probability of A times the probability of B .

Using the multiplication rule, you can also determine the probability of drawing two aces in a row from a deck of cards. The only way to draw two aces in a row from a deck of cards is for both draws to be favorable. For the firs draw, the probability of a favorable outcome is $4/52$. But because the first draw is favorable, only three aces are left among 51 cards. So the probability of a favorable outcome on the second draw is $3/51$. For both events to happen, you simply multiply those two probabilities together: $(4/52)(3/51) = (12/2652) = 0.0045$.

Note that these probabilities are not independent. If, however, you had decided to return the initial card drawn back to the deck before the second draw, then the probability of drawing and ace on each draw is $4/52$, as these events are now independent. Drawing and ace twice in a row, with the odds being $4/52$ both times, gives: $(4/52)(4/52) = (16/2704) = .0059$. In either case, you use the multiplication rule because you are computing probability for favorable outcomes in all events (Foroughbakhch & Badii, 2005).

Addition Rule. Given mutually exclusive events, finding the probability of *at least one* of them occurring is accomplished by adding their probabilities.

For example, what is the probability of one coin flip resulting in at least one head or at least one tail?

The probability of one coin flip landing heads is 0.5, and the probability of one coin landing tails is 0.5. Are these two outcomes mutually exclusive in one coin flip? Yes, they are. You cannot have a coin land both heads and tails in one coin flip; therefore, you can determine the probability of at least one head or one tail resulting from one flip by adding the two probabilities: $0.5 + 0.5 = 1.0$ (or certainty).

What is the probability of at least one spade or one club being randomly chosen in one draw from a deck of cards?

The probability of drawing a spade in one draw is $13/52$; the probability of drawing a club in one draw is $13/52$. These two outcomes are mutually exclusive in one draw because you cannot draw both a spade and a club in one draw; therefore, you can use the addition rule to determine the probability of drawing at least one spade or one club in one draw: $(13/52) + (13/52) = (26/52) = 0.50$

Non-mutually exclusive outcomes. For the addition rule to apply, the events must be mutually exclusive. Now consider the following example.

What is the probability of the outcome of at least one head in two coin flips?

Should you add the two probabilities as in the examples above? In the first example, you added the probability of getting a head and the probability of getting a tail because of those two events were mutually exclusive in one flip. In the second example, the probability of getting a spade was added to the probability of getting a club because those two outcomes were mutually exclusive in one draw. Now when you have two flips, should you add the probability of getting a head on the first flip to the probability of getting a head on the second flip? Are these two events mutually exclusive? No, they are not mutually exclusive. You can get an outcome of a head on one flip and a head on the second flip. So, because they are not mutually exclusive, you cannot use the addition rule, (if you did use the addition rule, you would get: $(1/2) + (1/2) = (2/2) = 1.0$, or certainty, which is incorrect. There is no certainty of getting at least one head on two flips.

Double counting. By using the addition rule in a situation that is not mutually exclusive, you are in fact “double counting”. One way of realizing this double counting is to use the classic theory of probability: list all different outcomes when flipping a coin twice, and assess the ratio of favorable outcomes (Table 2).

Table 2. An example of double counting.

First flip	with	Second flip
Head	+	Head
Head	+	Tail
Tail	+	Head
Tail	+	tail

There are 4 total outcomes. Three of the outcomes have at least one head; therefore, the probability of throwing at least one head in two flips is $\frac{3}{4}$ or 0.75, not 1.0. But if you had used the addition rule, you would have added the two heads from the first flip to the two heads from the second flip and gotten 4 heads in 4 flips, $\frac{4}{4} = 1.0$. But the two heads in that first pair constitute only one outcome and so by counting both heads for that outcome, you are “double counting” because this is the joint-occurrence that is not mutually exclusive (Foroughbakhch & Badii, 2005). To use the addition rule in a non-mutually exclusive situation, you must subtract any events that double account. In this case: $\frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4} = 0.75$. The notation, therefore, for at least one favorable occurrence in two events is: $P(A + B) = P(A) + P(B) - P(AB)$

This rule is read: The probability of at least one of the events A or B equals the probability of A plus the probability of B minus their joint occurrence (not that if they are mutually exclusive, then $P(AB)$ -the joint occurrence-equal 0, and you simply add the two probability. For example, what is the probability either a spade or an ace forma deck of card? The probability of drawing a spade is $\frac{13}{52}$; the probability of drawing an ace is $\frac{4}{52}$. But the probability of their joint occurrence (an ace of spades) is $\frac{1}{52}$. Thus,

$$P(A+B) = P(A) + P(B) - P(AB) = (\frac{13}{52}) + (\frac{4}{52}) - (\frac{1}{52}) = 0.3077$$

Conditional probability

Sometimes you have more information than simply total outcomes and favorable outcomes and, hence, are able to make more informed judgments regarding probabilities. For example suppose you know the following information.

In a particular village, there are 60 women and 40 men. Twenty of those women are 70 years of age or older; 5 of the man are 70 years of age or older (Table 3).

Table 3. An example of data for conditional probability.

Gender	> 70 years	≤ 69 years	Totals
Women	20	40	60
Men	5	35	40
Totals	25	75	100

What is the probability that a person selected at random in that town will be a woman? Because women constitute 60 % of the total population, the probability is 0.60.

What is the probability that a person >70 years of age selected at random will be a woman? This question is different because the probability of A (being a woman) given B (the person in question is >70 years of age is now conditional upon B (being >70 years of age). This is due to the fact that the probability of this latter question is $\frac{20}{25}$, or 0.80.

Conditional probability is found using the formula: $P(A/B) = AB/B = P(AB)/P(B)$, which is read: The probability of A given B equals the proportion of the total of A and B to the total of B . The vertical bar in the expression A/B is read given that or given (Foroughbakhch & Badii, 2005).

Probability distributions

A probability distribution is a pictorial display of the probability, $P(x)$, for any value of x . Consider the number of possible outcomes of two coins being flipped (Table 4).

Table 4. Probability distribution of two flipped coins.

Possible outcomes of two flipped coins	
H + H = 2 heads	
H + T = 1 heads	
T + H = 1 heads	
T +T = 0 heads	

Probability distribution (# of heads)	
x	$P(x)$
0	$\frac{1}{4} = 0.25$
1	$\frac{1}{2} = 0.50$
2	$\frac{1}{4} = 0.25$

Discrete vs. continuous variables

The number of heads resulting from coin flips can be counted only in integers (whole numbers). The number of aces drawn from a deck can be counted only in integers. These “countable” numbers are known as discrete variables: 0, 1, 2, 3, 4, and so forth. Nothing in between two variables is possible. For example, 2.6 is not possible. Qualities such as height, weight, temperature, distance and so forth, however, can be measured using fractions or decimals as well: 34.27, 102.26, and so forth. These are known as continuous variables.

Total of probabilities of discrete variables

The probability of a discrete variable lies somewhere between 0 and 1, inclusive. For example, the probability of tossing one head in two coin flips is 0.50. The probability of tossing two heads in two coin flips is 0.25, and the probability of tossing no heads in two coin flips is 0.25. The sum of probabilities for all values of x always equals 1. For example, in the two tables above, adding the probabilities for all the possible outcomes yields a sum of 1 (Feller, 1966; Goldberg, 1960).

The Binomial distribution

A discrete variable that can result in only one of two outcomes is called binomial. For example, a coin flip is a binomial variable; but drawing a card from standard deck of 52 is not. Whether a drug is either successful or unsuccessful in producing results is a binomial variable, as is whether a machine produces perfect or imperfect items (Foroughbakhch & Badii, 2005). Binomial experiments require the following elements:

1. The experiment consists of a number of identical events (n).

2. Each event has only one of two mutually exclusive outcomes. (These outcomes are called successes and failures).
3. The probability of a “success” outcome is equal to some percentage, which is identified as a proportion, π .
4. This proportion, π , remains constant throughout all events and is defined as the ratio of number of success to number of trials.
5. The events are independent.
6. Given all of the above, the binomial formula can be applied (x = number of favorable outcomes; n = number of events):

$$P(x) = n! / [x! (n - x)!] \pi^n (1 - \pi)^{n - x}$$
$$n! = n (n-1)(n-2) \dots (3)(2)(1)$$

Suppose a coin is flipped 10 times. What is the probability of getting exactly 5 heads? Using the binomial formula, where n (the number of events) is given as 10; x (the number of favorable outcomes) is given as 5; and the probability of landing a head in one flip is 0.5:

$$P(x) = n! / [x! (n - x)!] \pi^n (1 - \pi)^{n - x}$$
$$P(5) = [10! / 5!(5!)] 0.5^5 (1 - 0.5)^{10 - 5}$$
$$= 0.246$$

So, the probability of getting exactly 5 heads in 10 flips is 0.246, or 25%.

Mean and standard deviation.

The mean of the binomial probability distribution (μ) is determined by the formula: $\mu = n\pi$, where π is the proportion of favorable outcomes and n is the number of events. The standard deviation of the binomial probability distribution is determined by the formula: $\sigma = [n\pi, (1 - \pi)]^{1/2}$. For example, what is the mean and standard deviation for a binomial probability distribution for 10 coin flips of a fair coin? Because the proportion of favorable outcomes of a fair coin falling heads (or tails) is $\pi = 0.5$, simply substitute into the formulas: $\mu = n\pi = 10(0.5) = 5$, and $\sigma = [n\pi, (1 - \pi)]^{1/2} = \sigma = [10(0.5)(1 - 0.5)]^{1/2} = 1.58$ (Boomsma & Molenaar, 1994).

Sampling

The field of inferential statistics enables one to make educated guesses about the numerical characteristics of large groups. The logic of sampling provides a way to test conclusions about such groups only a small portion of its members. A population is a group of phenomena that have something in common. The term often refers to a group of people: examples are: all registered voters in a city, all members of the Barcelona Football Club, all the men who played golf at least once in the past year. But populations can refer to things as well as people: All items produced last Tuesday by the company, all daily maximum temperatures in July for major Mexican cities, and all basal ganglia cells from a particular species (Lloyd, 1967; Badii et al., 2000).

Often, researchers want to know things about populations, but don't have data for every person or thing in the population. If a company's customer service division wanted to learn

whether its customer were satisfied, it would not be practical (or perhaps even possible) to contact every individual who purchased one of their products. Instead, they might select a sample population. A sample is a smaller group of members of a population selected to represent the population. In order to use statistics to learn things about population, the sample must be random. A random sample is one in which every member of a population has an equal chance to be selected.

A parameter is a characteristic of a population. A statistic is a characteristic of a sample. Inferential statistics enables us to make an educated guess about population parameter based on a statistic computed from a sample randomly drawn from that population.

For example, suppose we want to know the mean income of the subscribers to a particular magazine- a parameter of a population. We draw a random sample of 100 subscribers and determine that their mean income is \$ 27,500 (a statistic). We conclude that the populations mean income μ is likely to be close to \$ 27,500 as well. This example is one of statistical inference.

Sampling distributions

Continuing with the above example, suppose that ten different samples of 100 people were drawn from the population, instead of just one. The income means of these ten samples would not be expected to be exactly the same, because of sampling variability. Sampling variability is the tendency of the same statistic computed from a number of random samples drawn from the same population to differ (Badii et al., 2000).

Suppose that the first sample of 100 magazine subscribers was “returned” to the population (made available to be selected again), another sample of 100 subscribers selected at random, and the mean income of the new sample computed. If this process were repeated ten times, it might yield the following sample means: 27,500, 27,192, 28,736, 26,454, 28,527, 28,407, 27,592, 27,684, 28,827 and 27,809.

These ten values are part of a sampling distribution. The sampling distribution of a statistic, in this case, of a mean, is the distribution obtained by computing the statistic for a large number of samples drawn from the same population (Taylor, 1961).

We can compute the sample mean of this sampling distribution by summing the ten sample means and dividing by ten, which gives a distribution mean of 27,782. Suppose that the mean income of the entire population of subscribers to magazine is \$ 28,000. We can infer that the first sample mean of \$27,500 was not a bad estimate of the population mean and that the mean of the distribution of ten sample mean, \$27,872, was even better. The more sample means included in the sampling distribution, the more accurate the mean of the sampling distribution becomes as an estimate of the population mean.

Random and systematic error

Two potential sources of error occur in statistical estimation, i.e., two reasons a statistic might misrepresent a parameter. Random error occurs as a result of sampling variability.

The ten sample means above differed from the true population mean because of random error. Some were below the true value; some above it. Similarly, the mean of the distribution of ten sample means was slightly lower than the true population mean. If ten more samples of 100 subscribers were drawn, the mean of that distribution, that is, the mean of those means, might be higher than the population's mean (Badii et al., 2000).

Systematic error or bias refers to the tendency to consistently underestimate or overestimate a true value. Suppose that the list of magazine subscribers was obtained through a database of information about air travelers. The samples that you would draw from such a list would likely overestimate the population mean of all subscribers' income because lower-income subscribers are less likely to travel by air and many of them would be unavailable to be selected for the samples. This example would be one of bias (Badii et al., 2000).

Central Limit Theorem

If the population of all subscribers to the magazine were normal, you would expect its sampling distribution of means to be normal as well. But what if the population were non-normal? The Central Limit Theorem states that even if a population distribution is strongly non-normal, its sampling distribution of means will be approximately normal for large sample sizes (over 30). The Central Limit Theorem makes it possible to use probabilities associated with the normal curve to answer questions about the means of sufficiently large samples.

According to the Central Limit Theorem, the mean of a sampling distribution of means is an unbiased estimator of the population mean. $\mu_m = \mu$. Similarly, the standard deviation of a sampling distribution of means is $\sigma_m = \sigma / \sqrt{n}$. The larger the sample, the less variable the sample mean. The mean of many observations is less variable than the mean of few. The standard deviation of a sampling distribution of means is often called the standard error of the mean. Every statistic has a standard error, which is a measure of the statistic's random variability. For example, if the population mean of number of fish caught per trip to a particular fishing pond is 3.2 and the population standard deviation is 1.8, what are the population mean and standard deviation of 40 trips? $\mu_m = 3.2$, and $\sigma_m = 1.8 / \sqrt{40} = 0.285$.

Properties of the normal curve

Known characteristics of the normal curve make it possible to estimate the probability of occurrence of any value of a normally distributed variable. Suppose that the total area under the curve is defined to be 1. You can multiply that number by 100 and say there is a 100% chance that any value you can name will be somewhere in the distribution. It should be noted that the distribution extends to infinity in both directions (Foroughbakhch & Badii, 2005).

Similarly, because half of the area of the curve is below the mean and half is above it, you can say that there's a 50% chance that a randomly chosen value will be above the mean and the same chance that it will be below it. It makes sense that the area under the normal curve is equivalent to the probability of randomly drawing a value in that range. The area is

greatest in the middle, where the “hump” is, and thins out toward the tails. That is consistent with the fact that there are more values close to the mean in normal distribution than far from it.

When the area of the standard normal curve is divided into sections by standard deviations above and below the mean, the area in each section is a known quantity. As explained above, the area in each section is the same as the probability of randomly drawing a value in that range. For example, .3413 of the curve falls between the mean and one standard deviation above the mean, which means that about 34% of all the values of a normally distributed variable are between the mean and one standard deviation above it. It also means that there is a .3413 chance that a value drawn at random from the distribution will lie between these two points.

Sections of the curve above and below the mean may be added together to find the probability of obtaining a value within (plus or minus) a given number of standard deviations of the mean. For example, the amount of curve area between one standard deviation above the mean and one standard deviation below is $.3413 + .3413 = .6826$, which means that exactly 68.26% of the values lie in that range. Similarly, about 95% of the values lie within two standard deviations of the mean, and 99.7% of the values lie within three standard deviations.

In order to use the area of the normal curve to determine the probability of occurrence of a given value, the value must first be standardized, or converted to a z- score. To convert a value to a z-score is to express it in terms of how many standard deviations it is above or below the mean. After the z-score is obtained, its corresponding probability may be looked up in a table. The formula to compute z-score is: $z = (x - \mu)/\sigma$, Where x is the value to be converted, μ is the population mean, and σ is the population standard deviation.

For example, a normal distribution of retail store purchases has a mean of \$14.31 and a standard deviation of 6.40. What percentage of purchases was under \$10? First, compute the z-score: $z = (x - \mu)/\sigma = (10 - 14.31) / 6.40 = -0.67$. The next step is to look up the z-score in the table of standard normal probabilities. The standard normal table lists the probabilities (curve areas) associated with given z-scores. The answer is that about 25% of the purchases were under \$10. If you had wanted to know the percentage of purchases above a certain amount? Approximately 75% of the purchases were above \$10.

Normal approximation to the binomial distribution

Some variables are continuous that there is no limit to the number of times you could divide their intervals into still smaller ones, although you may round them off for convenience. Examples include age, height, and cholesterol level. Other variables are discrete, or made of whole units with no values between them. Some discrete variables are the number of children in a family, the sizes of televisions available for purchase, and the number of medals awarded at the Olympic Games.

A binomial variable can take only two values, often termed *successes* and *failures*. Examples include coin flips that come up either heads or tails, manufactured parts that

either continue working past a certain point or do not, and basketball tosses that either fall through the hoop or do not.

The outcomes of binomial trails have a frequency distribution, just as continuous variables do. The more binomial outcomes there are (for example, the more coins you flip simultaneously), the more closely the sampling distribution resembles a normal curve. You can take advantage of this fact and use the table of standard normal probabilities to estimate the likelihood of obtaining a given proportion of successes. You can do this by converting the test proportion to a z-score and looking up its probability in the standard normal table (Foroughbakhch & Badii, 2005).

The mean of the normal approximation to the binomial is $\mu = n \pi$, and the standard deviation is $\sigma = [n\pi, (1 - \pi)]^{1/2}$, where n is number of trails and π is the total number of successes in the population divided by n. The approximation will be more accurate the larger the n, and the closer the proportion of successes in the population to 0.5. For example, assuming an equal chance of a new baby being a boy or a girl ($\pi=0.5$), what is the likelihood that more than 60 out of the next 100 births at a local hospital will be boys? $z = (x - \mu)/\sigma = (60 - (100(0.5)) / [100(0.5)(1-0.5)]^{1/2} = 10/5 = 2$. Based on the normal curve Table, a z-score of the 2 corresponds to a probability of 0.9772. Then, there is a 0.9772 chance that there will be 60% or fewer boys, which means that the probability that if the assumption that the chance of a new baby being a girl is the same as it being a boy is correct, the probability of obtaining 60 or fewer girls in the next 100 births is also 0.9772 (Foroughbakhch & Badii, 2005).

Conclusions

We live in a world that is governed by the rules of probability. In fact, only two events, i.e., death and tax payment are deterministic events where there is no escape from them. Decision making with regards to any other event, phenomenon, process, object, etc. is associated with probability of committing error. The point in question is to be able to estimate the level of this error and preferably place a range within which we are able to locate the probability of finding the parameters under study. Therefore, a grasp of the fundamentals of probability theory is of paramount relevance to all who do experimental sciences.

References

1. Badii, M.H., A.E. Flores, R. Foroughbakhch, & H. Quiróz. Fundamentos de muestreo. Pp. 129-144. In: M.H. Badii, A.E. Flores & L.J. Galán (eds). Fundamentos y Perspectivas de Control Biológico. UANL. Monterrey.
2. Boomsma, A., & I.W. Molenaar. 1994. Four electronic tables for probability distributions. Amer. Statist. 48: 153-162.
3. Feller, W. 1966. An Introduction to Probability Theory and its Applications. 2nd Ed. John Wiley & Sons Inc., N.Y.
4. Foroughbakhch, R., & M.H. Badii. 2005. Métodos Analíticos Estadísticos. UANL. Monterrey. Goldberg, S. 1960. Probability: An Introduction. Prentice Hall, Inc., Englewood Cliffs, N.J.
5. Hodges, J., & E. Lehman. 1964. Basic Concepts of Probability and Statistics. Holden Day Inc., San Francisco. Hoel, P.G. S.C. Port, & C.J. Stone. 1972. Introduction to Probability Theory. Houghton

6. Kemeny, J.G., H. Mirkil, J.L. Snell, & G.L. Thompson. 1958. Finite Mathematical Structures. Prentice Hall, Inc., Englewood Cliffs, N.J.
7. Lloyd, M. 1967. Mean crowding. J. Anim. Ecol. 36: 1-30.
8. Mehta, C.R., & J.F. Hilton. 1993. Exact power of conditional and unconditional tests. Amer. Statist. 47: 91- 98.
9. Mosteller, F., R. Rourke, & G.Thomas. 1961. Probability and Statistics. Addison-Wesley Publishing Co., Inc.,Reading, Mass.
10. Parzen, E. 1960. Modern probability Theory and its Applications. John Wiley & Sons Inc., N.Y.Taylor, L.R. 1961. Aggregation, variance and the mean. Nature, 189: 732-735.
11. El Dr. Mohammad Badii es Profesor e Investigador de la Universidad Autónoma de Nuevo León. SanNicolás, N. L., México, 66450.
12. El Dr. Jorge Castillo es Profesor e Investigador de la Universidad Autónoma de Nuevo León. San Nicolás, N. L., México, 66450.
13. El Dr. R. Foroughbakhch es Profesor e Investigador de la Universidad Autónoma de Nuevo León. SanNicolás, N. L., México, 66450.
14. El Dr. Klender Cortez es Profesor e Investigador de la Universidad Autónoma de Nuevo León. San Nicolás.
15. N. L., México, 66450.