Reverse Engineering as a Sustainable Process

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Abstract

Reverse engineering mimics key aspects of an existing object to generate precise or enhanced virtual/physical representations. This method is valuable for capturing the digital essence of an object and typically involves advanced equipment, computing resources, and expert skills. However, for sustainability, it's preferable to reduce reliance on such resources.

This study proposes a sustainable reverse engineering process comprising five steps. Initially, the object is described using basic geometric shapes. Next, these shapes are represented through simulated planner point clouds. The subsequent step involves merging these point clouds to create coordinated ones.

Using an off-the-shelf CAD package, a solid model is then generated in the fourth step. Finally, a 3D-printed prototype is produced from the CAD model. Compared to traditional scanned point cloud-based reverse engineering, this approach is less computationally intensive, yields relatively accurate models, and operates at a faster pace. Integrating this method into product life cycle engineering can foster sustainable product development.

Keywords: Reverse Engineering; Sustainability; Product Development; Product Life cycle, Human Cognition; Geometric Modeling

1. Introduction

According to the United Nations (UN), sustainability means achieving current needs without jeopardizing the potential of fulfilling future needs [1]. Seventeen goals, known as the Sustainable Development Goals (SDGs), have been introduced to ensure sustainability [2].

Goal 12 (ensure sustainable consumption and production patterns) is highly relevant to the product/system life cycle. Two of the remarkable sub-goals of goal 12 are as follows: 1) By 2030, achieve sustainable management and efficient use of natural resources. 2) By 2030, substantially reduce waste generation through prevention, reduction, recycling, and reuse.

Whether or not these sub-goals are achieved can be ensured by measuring the indicators called material, energy, and product efficiencies [3,4]. Nevertheless, other indicators can be considered to make sustainability more meaningful. In this case, an indicator defined as system efficiency is considered in this study. The description of this indicator is as follows.

A set of systems (System X_1,…) is needed to carry out the activities associated with the life cycle stages, as shown in Fig. 1. The systems can work independently or concurrently. However, if the systems are expensive and require highly sophisticated computational procedures and highly skilled human resources, the stakeholders might encounter unwanted complexity.

As a result, the activities needed to support the product life cycle stages may not be performed as expected, resulting in unnecessary wastage of time and resources. These problems can be measured quantitatively or qualitatively using an indicator denoted as system efficiency. Therefore, low system efficiency prevents smooth executions of the activities associated with the life cycle stages. How to achieve high system efficiency is thus an important aspect of sustainability or product life cycle. This study addresses this issue.

Particularly, system efficiency from the perspective of reverse engineering [5] is considered, which is directly associated with the stages called strategies, customer needs, concepts, virtual model, selected model, and prototypes (Fig. 1). In "conventional engineering," multiple concepts of a product are first generated based on the customer needs and strategy of the relevant organization. The concepts are then converted into product models (virtual models). Subsequently, the optimal virtual models are converted into physical models (prototypes) before going for manufacturing. Thus, conventional engineering refers to the pathway "concept-model-object." Many systems are used to supports this pathway. Sometimes an opposite pathway is selected called reverse engineering [5]. As the naming suggests, reverse engineering refers to the pathway "object-model-concept". Thus, in reverse engineering, the valuable information extracted from an existing object is utilized to create a model. From the model, the concept underlying the object is extracted. The above description reverse engineering is somewhat a classical description. In reality, reverse engineering takes a somewhat different form. It mimics some of the critical geometric features of an existing object, creates its accurate or enhanced virtual/physical models, and generates digital footprints of the object, as shown in Fig. 2. In most cases, scanned point clouds are used as input information, as seen in Fig. 2. A review of real-life reverse engineering is given in the next section. As described in Section 2, conventional reverse engineering requires sophisticated devices and computing facilities. From the viewpoint of system efficiency, a reverse engineering process that is less dependent on sophisticated devices and complex computations exhibits can exhibit high system efficiency and become more sustainable. From the above consideration, the article is written.

The rest of this article is organized as follows: Section 2 presents a literature review. Section 3 outlines the steps of the proposed sustainable reverse engineering process. Section 4 presents two case studies implementing the proposed reverse engineering process. T This section also compares results with the traditional scanned point cloud-based reverse engineering, exhibiting the proposed reverse engineering's efficacy. Section 5 concludes this article.

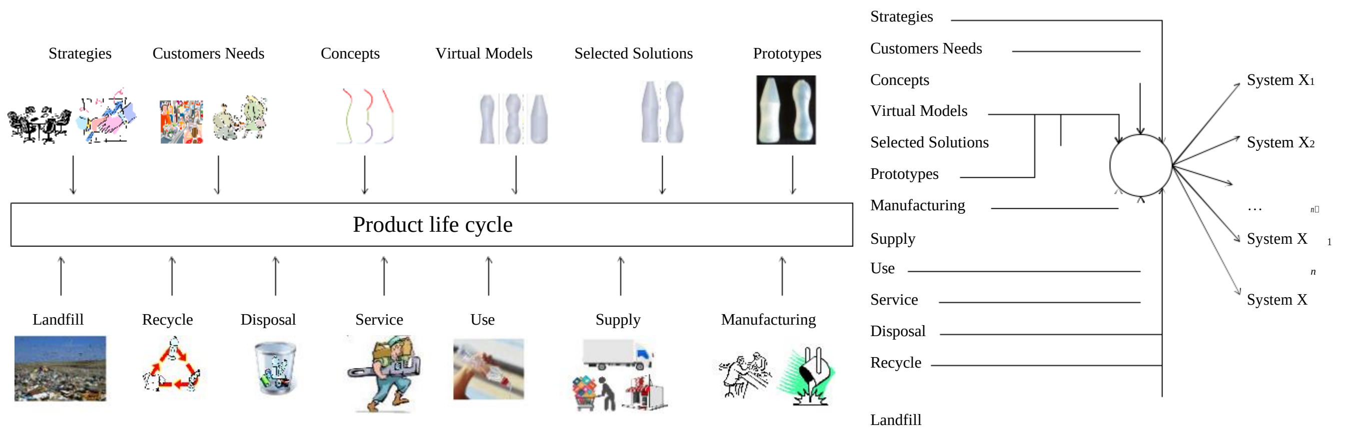

Fig. 1. Product life cycle and supporting systems.

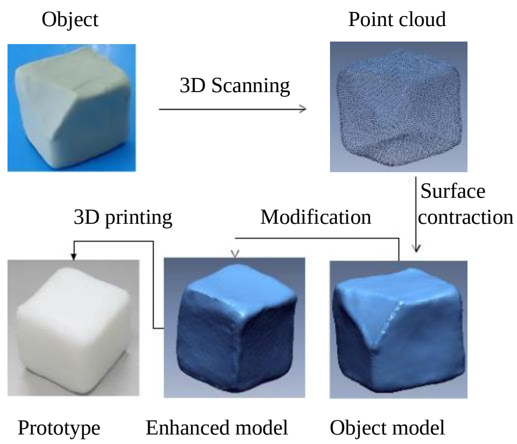

Fig. 2. Conventional reveres engineering.

2. Related Work

This section reviews the relevant literature describing various types of real-life reverse engineering techniques and their application areas.

Buonamici et al. [6] surveyed the scanned point cloud-based reverse engineering (Fig. 2) and reported its advantages and limitations. According to this survey, a reverse engineering process depends on the solid model (virtual model) reconstruction process using mostly off-the-shelf CAD packages. This process starts with point cloud data acquisition and terminates at virtual model construction. The typical intermediate steps are preprocessing, segmentation, feature classification, and mesh/surface fitting. The remarkable thing is that the laser-scanned point clouds exhibit noise and outliers. As a result, preprocessing and post-processing are required before extracting the geometric features. Numerous authors delved into the preprocessing and post-processing of a scanned point cloud. For the relatively small-size point clouds, one of the well-known preprocessing techniques is the iterative closest point algorithm [7, 8]. Many authors have applied it in real-life reverse engineering. For example, Guan and Gu [9] used both iterative closest point algorithm and chord deviation calculation to eliminate the noisy points before surface reconstruction of the bowl and blade of a ship. However, the relatively large-size point clouds are treated differently. Particularly, feature extraction from a large-spatial point cloud has been an issue for creating digital footprints of buildings and relevant systems. For example, Araújo and Oliveira [10] developed a fast cylinder-detection technique from a large point cloud (point cloud of a chemical plant). The detection technique employs a linear-time circle-detection algorithm. The algorithm projects the point cloud onto a set of uniformly-distributed directions defined over the unit hemisphere. It calculates points whose normal vectors are approximately perpendicular to a given projection and refines these directions. The extracted cylindrical surfaces are obtained by fitting a cylinder to each connected component that passes a validity test. Moritani et al. [11] articulated a method to preprocess a laser-scanned point cloud of an arbitrary space to extract the digital footprint of a piping system for renovating it (piping system). The remarkable thing is that the simulated point cloud of the core feature (a cylinder) must interact with the scanned point cloud while removing the noise and outliers. This is achieved by employing a sophisticated algorithm, namely, iteratively reweighted least-squares method. Massafra et al. [12] developed a method of dealing with a large point cloud for feature extraction. This method first divides a massive point cloud into small feature-based points cloud for constructing the digital footprints of wooden trusses of a historical building. Finally, parametric modeling was added to refine the objects. Other computer-aided engineering techniques can aid the refinement process. This way, more effective building information modeling systems can be constructed. In order to develop a building facility's digital model, a semi-automatic method was developed by Zeng et al. [13]. In this method, first, a pre-trained deep neural network extracts a 50-dimensional feature vector for each point. This helps segment the point clouds in clusters where the region-growing algorithms are used. The user-selected visualized features are then used as examples to run the peak-finding algorithm to determine positive matches.

In addition to scanned point clouds, imaging plays a vital role in reverse engineering. For example, Mengoni and Leopardi [14] summarized the performances of the hardware and software tools used in both active sensor or range-based technology (3D laser scanner) and Structure from Motion (SfM) based technology reverse engineering applied to digital documentation of archeological heritage. The former one directly acquires the coordinates of the exposed surfaces of an object. These coordinates are converted into a 3D solid model after performing the necessary pre- and post-processing, as mentioned before. On the other hand, the latter (SfM) combines a series of photos of an object and produces its 3D solid model. SfM is good at constructing an object's facsimile. When the object is scaled, high-resolution point clouds obtained by using a 3D scanner must be integrated with the SfM technique to ensure the accuracy of the object. In addition to SfM, other imaging techniques can be used for reverse engineering. For example, multiple spin-image can be used to reconstruct a voxel-based digital model of an object, as shown by Nanya et al. [15]. The voxels are generated from the images. This method requires a manual check on some edges, ensuring the targeted edge is either inside or outside the desired region of the image. This results in the elimination of redundant voxels. Reconstruction of silhouetted concave regions and extremely thin segments are two of the main challenges of the method. To overcome these challenges, the hardware/software used in data acquisition (imaging) needs improvement.

As far as quality control is concerned, reverse engineering has been very instrumental. Zachos et al. [16] showed how to use the point cloud of a manufactured turbine blade to determine its accuracy compared to its design. Segreto et al. [17] developed a point cloud-based reverse engineering technique useful for quality control of aerospace components made of carbon fiber-reinforced polymer composites. The study showed that the comparison between 3D digital models of the manufactured part and its design requires dedicated user-interfaces. Urata et al. [18] developed a reverse engineering method to produced high surface finished casted products. In this method, first, an existing casted part is scanned to prepare its surface mesh data. The data are processed using a customized geometric modeling technique to separate the as-cast and machined surfaces. After that, the boundary is modified to create a digital model of casting. This way, high-quality mass production using casting can be achieved. Yang et al. [19] used reverse engineering in welding quality control. They have developed a weighted neighborhood search algorithm to delete points from the point cloud of a welded part and extract the points representing a subtle feature (e.g., welded sections of a spherical object). The point cloud elimination process requires a complex computational procedure based on a multi-threshold weighting adjustment calculation customized for the targeted features.

When an object is not available, sketches can be used to carry out reverse engineering, as sketches are a natural and intuitive means to express concepts [20]. In this case, a sketch must be converted into an off-the-shelf CAD compatible solid model. Different types of machine learning algorithms are used to reverse engineer an object from its sketch [20]. In these algorithms, the topological rule-bases of some selected vertex-edge pairs are used. These rules must establish the topological relationships among the edges and vertices at the rough intersections of a sketch. The rules thus help interpret a rough sketch in terms of well-formed geometrical entities. The rules are often user-defined and categorized in terms of features (e.g., rectangular box, cylindrical shape, through holes, and alike). For example, Tanaka et al. [21] developed sex sets of rules to develop a system. The system converts a rough sketch (composed of curved and straight edges) into a CAD package compatible solid model.

In addition to scanned point clouds, images, and sketches, there is another type of reverse engineering where analytical points are used to mimic an object's important features. The analytical point clouds can be created in different ways. For example, Montlahuc et al. [22] presented an analytical point cloud-based reverse engineering method. The method analytically creates as-scanned point clouds of industrial assembly models. It first generates a watertight triangle mesh wrapping the relevant CAD model. The method finally converts the mesh into a point cloud. Since the method does not use a laser scanner to obtain the point cloud, it is free from computational complexities (redundant and noisily points elimination, surface construction from the point cloud, and alike). This does not mean that it is free from all sorts of computational complexity. Notably, how to control the distribution of points, remove the hidden points, and adjust the misalignments are some of the computational challenges of the method. Tashi et al. [23-25] developed an analytical point cloud-based reverse engineering. In this case, a recursive process is used to create a planner point cloud. The process is originated from work done by Ullah et al. [26]. It is effective in reverse engineering 2.5 and 3D shapes. This method helps develop a new breed of reverse engineering defined as human cognitive reverse engineering [27].

3. Method

The proposed sustainable reverse engineering process is schematically illustrated in Fig. 3. The process integrates humans and machines to achieve its goal and discourages scanned point clouds' direct involvement. Rather it employs planner point clouds simulated by analytical approaches.

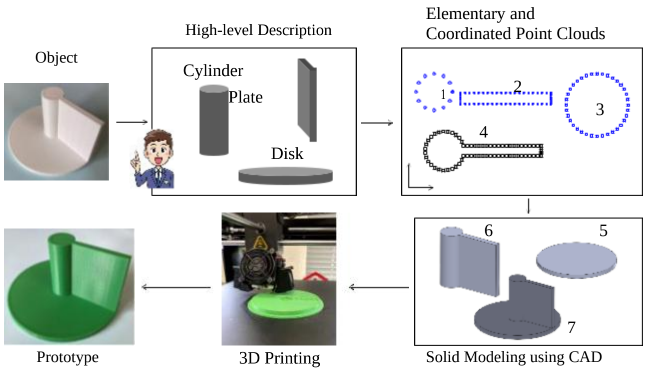

Fig. 3. The proposed sustainable reverse engineering method.

As seen in Fig. 3, the process consists of five major steps. In synopsis, the first step describes a given object using some elementary geometric shapes. The second step represents the elementary geometric shapes using some simulated planner point clouds. The third step creates coordinated point clouds by combining some of the elementary point clouds. The fourth step creates a virtual model of the object using an off-the-shelf CAD package where the input information is some selected elementary and coordinated point clouds. The fifth step creates a 3D printed prototype using the virtual model. The descriptions of the steps are as follows:

As mentioned above, the first step creates a high-level description of the object to be reverse-engineered. The output of this step depends on the individual who describes the object using some linguistic expressions. For example, according to the perception of an individual familiar with the basic geometrical shapes, the object shown in Fig. 3 consists of three elementary shapes named cylinder, plate, and disk.

The second and third steps create the elementary and coordinated point clouds, respectively, representing the planner features of the shapes. For creating the elementary point clouds, the algorithm shown in [23-26] can be used. For example, the three elementary point clouds shown in Fig. 3 are created using the algorithm shown in [23-26]. The point clouds denoted as 1, 2, and 3 represent the cross-sectional areas of the elementary shapes called cylinder, plate, and disk, respectively. For the sake of solid modeling in step 4, some point clouds can be integrated to create the coordinated point clouds. For example, in Fig. 3, the point clouds 1 and 2 are integrated to create the coordinated point cloud denoted as 4. The coordinated point cloud maintains at least the C^0 continuity among its constituents. Thus, after completing steps 2 and 3, the reverse engineering process is populated with both elementary and coordinated point clouds.

The fourth step creates a solid model of the elementary and coordinated point clouds using an off-the-shelf CAD package. For example, as shown in Fig. 3, the point clouds of 3 and 4 are converted into two sloid models denoted as 5 and 6, respectively, using a commercially available CAD package. (The name is not disclosed to avoid commerciality.) It is worth mentioning that most commercially available CAD packages nowadays offer a function to directly import the coordinates of the points comprising a point cloud. The imported point clouds can be converted into (most likely) closed curves/lines before performing solid modeling using some predefined functions, e.g., rotation, extrusion, loft, sweep, shell, and alike, see https://web.mit.edu/2.972/www/solid_ modeling.html). The solid models of the elementary point clouds and coordinated point clouds can be added to create the object's virtual model to be reverse-engineered. For example, as shown in Fig. 3, solid model 7 is the virtual model of the real object created by adding solid models 5 and 6.

The last step is 3D printing. In this step, a suitable additive manufacturing process can create the prototype using the virtual model's information. For example, the prototype shown in Fig. 3 is created using a commercially available 3D printer where the input information is model 7 (Fig. 3).

4. Results and Discussions

This section presents two case studies to elucidate the efficacy of the proposed sustainable reverse engineering.

The first case study deals with the reverse engineering of the object shown in Fig. 3. The reverse engineering of the object using the proposed process is already presented in the previous section (Fig. 3). This section shows its comparison with conventional reverse engineering. The results are shown in Fig. 4. As seen in Fig. 4, when the object is scanned to create its point cloud, it exhibits noise and outliers at the intersecting edges. As a result, the virtual model is quite inaccurate. This is reflected in the 3D printed prototype. On the other hand, the virtual model and prototype created using the presented sustainable reverse engineering process are free from shape inaccuracies (Fig. 3).

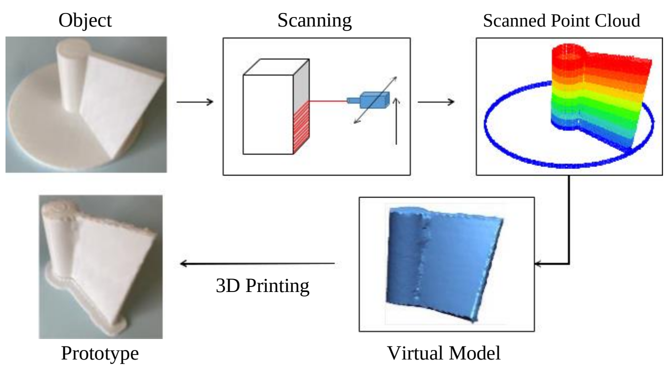

Fig. 4. Conventional reverse engineering of the object shown in Fig. 3.

The other case study is shown in Figs. 5 and 6. In particular, Fig. 5 shows the result when conventional reverse engineering is applied. In this case, the object (relatively complex compared to the object shown in Fig. 3) is scanned using a laser scanner. The point clouds result in inaccurate virtual models, and the inaccuracy persists even when the noise and outliers are removed. The degree of inaccuracy this time (Fig. 5) is severe than the previous case (Fig. 4). For this reason, it is not printed.

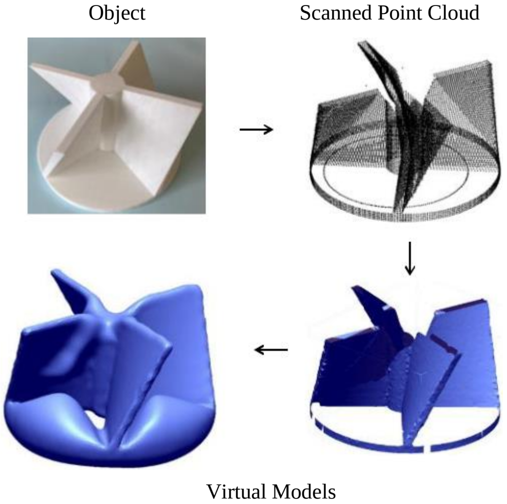

Fig. 5. Conventional reverse engineering of relatively complex object.

When the object shown in Fig. 5 reverse engineered using the proposed reverse engineering process, the results improve, as shown in Fig. 6. As seen in Fig. 6, the object's high-level description (the object consists of the shapes called cylinder, plate, and disk) remains the same, but its elementary and coordinated point clouds are somewhat different. Particularly, point clouds 1, 2, and 3, representing a circular arc, rectangular boundary, and circle, respectively, constitute the elementary point clouds. The coordinated point clouds 5, 6, and 7 are created by rotating the other coordinated point cloud, point cloud 4, preserving the object's symmetry. The coordinated point cloud is created, adding the elementary point clouds 1 and 2 and subtracting one of the lines in the point cloud 2. The coordinated point clouds 4, 5, 6, and 7 can be added, maintaining C0 continuity to create another coordinated point cloud denoted as 8, as shown in Fig. 6. The point cloud can be rotated to create the last coordinated point cloud denoted as point cloud 9. The point clouds 8 and 9 can be imported to an off-the-shelf CAD package to perform geometric modeling. The point cloud to solid model (model 11) conversion process is shown by entity 10 where two planar boundaries (point clouds 8 and 9) separated by the desired height are extruded to create the solid model (model 11). Thus, the solid modeling this time creates models 11 and 12. These two models can be added to create the virtual model (denoted as 13). The virtual model can be printed using an available 3D printer for manufacturing the prototype, as shown in Fig. 6. Therefore, the presented reverse engineering produces better results than the scanned point cloud-based conventional reverse engineering.

To be more specific, a quantitative comparison between the conventional and proposed reverse engineering is given in Table 1. Five parameters are used for the sake of comparison, namely, scanning time (min), size of the point clouds (number of points), point cloud editing time (min), geometric modeling time (min), and shape accuracy. The scanning time measures time to scan an object using a commercially available scanner under the standard scanning conditions. For the object shown in Figs. 5 and 6, the scanning time is about 11 minutes. The point clouds' size for conventional reverse engineering (Fig. 4) is 15164 points, whereas it is only 980 points (models 8 and 9 in Fig. 5) for the proposed method. Point cloud editing time means removing noise and outliers using the standard functions available to a scanning-based revere engineering system. In contrast, for the proposed method, it is the time necessary to complete the operations in the boundary "Elementary and Coordinated point clouds" (Fig. 6). For the conventional method, it is about 30 minutes, whereas, for the proposed method, it is about 10 minutes. After performing point cloud editing under standard conditions, geometric modeling time for conventional reverse engineering means the time needed to create a solid CAD model. On the other hand, the proposed method, the geometric modeling time means the time needed to perform geometric modeling in the boundary "Solid Modeling using CAD" (Fig. 6). Geometric modeling time for conventional reverse engineering is about 1 minute, whereas it is about 5 minutes for the proposed method. The proposed method outperforms the conventional one (compare the virtual models in Figs. 5 and 6) regarding shape accuracy. In synopsis, the proposed reverse engineering method's system efficiency is much better than that of the conversational one.

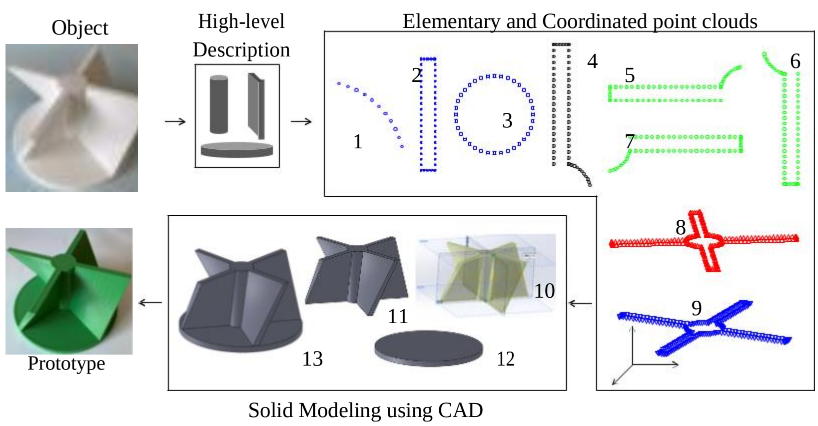

Fig. 6. The other application of the proposed sustainable reverse engineering.

Table 1. Comparison between conventional and proposed reverse engineering.

Reverse engineering	Scanning time [min]	Size of point clouds [number of points]	Point cloud editing time [min]	Geometric modeling time [min]	Shape accuracy
Conventional	11	15164	30	1	Not accurate
Proposed	0	980	10	5	Accurate

5. Concluding Remarks

The activities associated with the early stages of a product life cycle (i.e., the stages of strategies, customer needs, concepts, virtual model, optimal model, and prototypes) can be performed without sacrificing system efficiency if the proposed reverse engineering is used. The method is free from computational and systemic complexity compared to the scanned point cloud-based conversational reverse engineering. The next phase of this study will study more complex objects having a complex twisted radius of curvature.

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