

**AN OVERVIEW ON “APPLICATION OF MATHEMATICAL TOOLS AND AI & ML IN  
IMAGE PROCESSING”**

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### **Abstract**

*In today's world, secure communication often involves transmitting sensitive messages through images, videos, and signals over long distances. The images are produced in different ways, photography, digital camera images, scanned images using CT, MRI, and ultrasound images. Automating or simplifying the assessment and interpretation of images is image processing's ultimate goal. The normally processing the images are edges, corners, smooth color regions, textures and coherent regions. Mathematics serves as the guiding force behind every pixel transformation and enhancement. Due to these mathematical tools usage play a significant role in modifying images. Image processing includes tasks like enhancement of the image, its restoration and its segmentation. Numerous fundamental image processing tasks, such as edge detection, segmentation, enhancement, de-noising, and de-blurring, are closely linked to mathematics. Most commonly used De-Blurring, De-noising, etc. The different mathematical models such as histogram equalization, spatial filters & Laplacian distribution are discussed. In this book chapter the cloud computing using image processing is discussed.*

**Keyword:** *Image Processing, Artificial Intelligence, Mathematical methods, Cloud computing*

### **Introduction**

The recent communication systems using satellite, the image recorded in space and the recorded images are sent to control station in ground state, it is not cleared and blurred image. The increase in communication system faces a great challenge for better resolution, fast image transmission and storage systems. In current situation, increased integration of the fields of image processing incorporate recent technical tools which are used to edit the images received from satellite communication. In this position, the mathematical tools, digitalized processing is used in image processing to get clear cut images of satellite. Sometimes in digital image processing, Issues may arise from the conversion of pictures to digital data, calibration, the elimination of distortion and noise accumulation during processing, and other related factors. As the digital images are presented in two dimensional formats, their processing can be formulated in multi-dimensional systems. We can model digital image processing as a multi-dimensional system can be modeled as multidimensional systems since images are defined over two dimensions. Image processing domains and the mathematics have been more integrated recently. <sup>1</sup>

Processing a digital image finds distinct advantages compared to processing an analogue image due to the development of digital computers. With the advancement of mathematics and computers, as well as the need for a wider range of practical solutions, digital image processing has grown quickly. Processing digital images with a digital computer is referred to as digital image processing.<sup>2</sup> A limited number of components, including picture elements, image elements, pixels, and pixels, make up digital image processing. In public spaces, artificial intelligence (AI) is being used to advance face recognition and security features, such as identifying and tracking intruders, objects, and patterns in real-time photos and videos. AI delves into techniques related to image augmentation, cloud computing, digital watermarking, machine learning, neural networks,

optimization algorithms, and data pre-treatment techniques.<sup>3</sup> This chapter covers the fundamentals of digital watermarking, image security, cloud computing, picture augmentation, formats of representation, techniques for image enhancement, image filtering, optimization techniques, machine learning, neural networks and data pre-treatment.

### **Role of Mathematics in Image Processing**

Numerous mathematical techniques are employed, such as the Fourier transformations, low & high-pass filters, zero crossings in the Laplacian, 1<sup>st</sup> and 2<sup>nd</sup> order partial derivatives, gradient, the Laplacian, discrete approximations by finite differences, averaging filters, order statistics filters, and convolution, for the spatial domain of image filtering and the frequency domain of the Fourier transform, zero crossings of the Laplacian, etc. Accurate identification and use of mathematical approaches and algorithms is necessary for effective image processing.<sup>4</sup> The degree to which the mathematical method is suited for a given problem-solving scenario determines the overall efficacy of the image processing procedure. It's hard to determine which of the several methods devised to process these digital photographs looks comparatively more natural. The field of image processing has a relatively longer history. The earliest approaches may have originated from techniques to process 1-D signals. More advanced instruments have been created. Stochastic modelization, techniques using partial differential equations and wavelets are the three primary avenues that emerge. A common foundation for stochastic modelization is the notion of Markov random fields. It directly addresses digital photos. The theory of Wavelet is derived from the science of signal processing and is rooted with PDE-based decomposition techniques. Any digital image processing procedure starts with these basic stages.<sup>5</sup> Table 1. describes the different stages involved in image processing.

**Table 1. Stages of Image Processing**

<b>Stages involved in image processing</b>	<b>Illustration</b>
Image Accession	The image is collected from a camera and digitized (if the camera is not a digital camera) using an A-D (analog to digital) converter for advanced processing using a computer.
Image Improvement	This stage involves suitable modification of the image for a particular purpose. These methods are mainly used to focus the subtle or important aspects of the image, such as adjustments in brightness and contrast. The image enhancement process is relatively a subjective process. <sup>6</sup>
Image Reconstruction	This step is to focus on picture appearance enhancement. This stage is objective because it is linked to either a mathematical or a probability model to explain why the image is deteriorated. Processes like noise reduction or blur reduction etc are the examples.
Color Image Procedure	This stage is intended to process colored images, like 16 bit RGB pictures or RGBA pictures. Modelling the color and color corrections are taken up in this stage. <sup>7</sup>
Wavelets and Processing at Multiple Resolutions	Wavelets are the units of different resolutions in which the pictures are represented. Subdivision of images into smaller sets facilitates representation in pyramidal formats and compression of data.
Image Simplification	The limitation of data storage and transfer require images to be compressed in data size. This is very essential for handling images over internet. For example, a tiny thumbnail of a picture we see on google is severely compressed variant of the original picture. One can access the actual resolution only when the thumbnail is clicked. The bandwidth of the server is conserved using this compression or simplification procedure.
Phenotypic Analysis	The downstream processes like representation or description of shapes of the image need separation of image components. Morphological techniques,

	which are a set of mathematical techniques at the core, are used in this stage. The means to achieve this are provided by morphological processing, which is basically a set of mathematical procedures. For example, erosion and dilation techniques may be used to sharpen or blur the edges in the picture. <sup>8</sup>
Image Dividing	This stage is called image segmentation. The image is segmented into several components to facilitate representation in some easily understandable and straightforward formats. Segmenting the image enhances the effectiveness of automated analytical models by facilitating computers to focus on the more important portions of the picture and ignore the not significant portions. <sup>9</sup>
Depiction and Synopsis	After segmentation stage, question arises as to whether to represent the segment as a boundary or as an entire region. The process of finding the attributes that give quantified information which is needed to separate one class from the other is called description. <sup>10</sup>
Identification and Detection of Objects	The automated system should identify the segmented part from the picture and label it like “vehicle” or “dog” to alert human users. This stage is called representation and detection. <sup>11</sup>

### Digital Image Representation

A key component of image processing is digital image representation, which dictates how computers store, process, and interpret images. Arrays of pixel values are commonly used to represent images, each pixel representing a distinct place within the image. Different kinds of graphics are shown by distinct colour models, such as CMYK (cyan, magenta, yellow, and black) and RGB (red, green, and blue). A digital image is a limited set of discrete samples, or pixels, of anything that may be seen. Each pixel has a unique discrete value within a finite range, and depicts a two dimensional or even higher "view" of the item. The magnitude of visible light, infrared light, x-ray absorption, electron absorption, and such other measurable parameters, such as ultrasonic wave impulses, can all be represented by the pixel values. It is adequate for the samples to form a spatial structure with two-dimensions that can be represented as a picture; the picture is not required to possess any meaning visually. The image could be captured using any of the gadgets like digital camera, a scanner, an electronic microscope, an ultrasonic stethoscope, or any other optical or non-optical sensors.<sup>12</sup>

Creation of a Digital Image

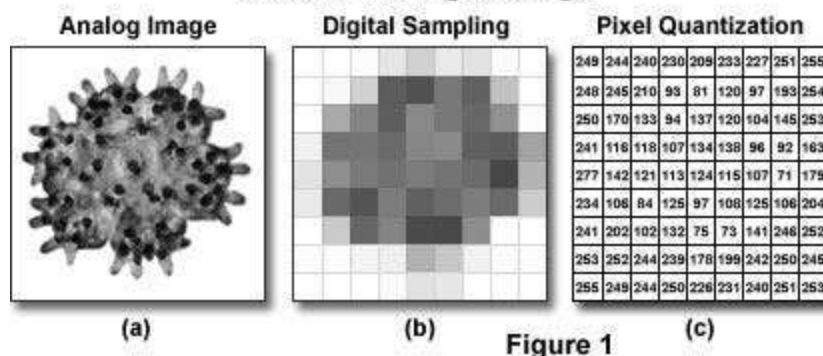


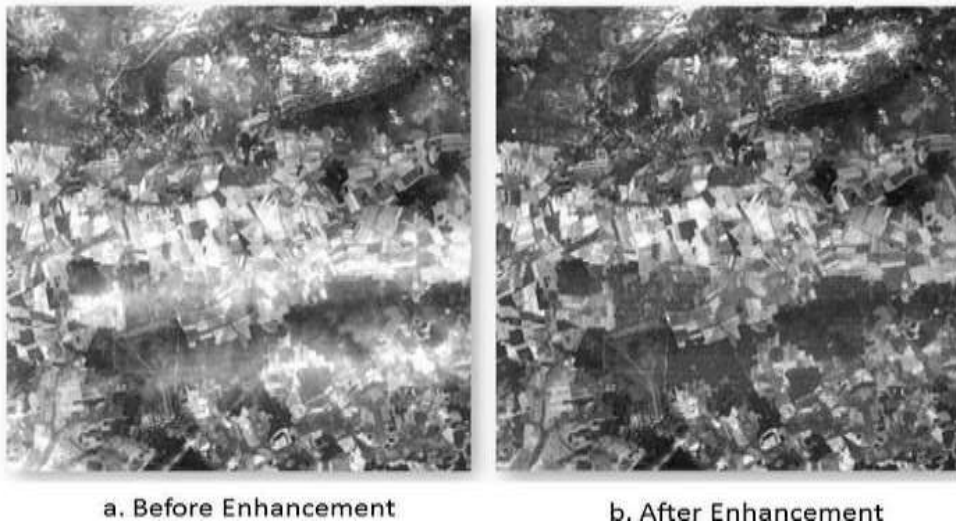
Fig. 1. Digital Image Representation (image search: Digital image representation inside a computer)

### Image Enhancement Techniques

Using techniques like noise reduction or feature emphasis, image enhancement improves an image's visual quality. These methods can include sharpening, which improves object features and edges, and contrast enhancement, which makes an image's contrast between light and dark areas greater.

Methods in spatial domain methods in frequency domain are the two basic categories into which image-enhancing techniques may be divided. The plane of the image is called as the "spatial domain," and methods in this area are predicated on directly modifying an image's pixels. The basis

of frequency domain processing methods is altering an image's Fourier transform. When compared to an unenhanced image, an improved image has greater contrast and more information. Applications for image enhancement are excellent.<sup>13</sup> It is applied to improve images obtained from satellites, remote sensing equipment, medical imaging, etc.

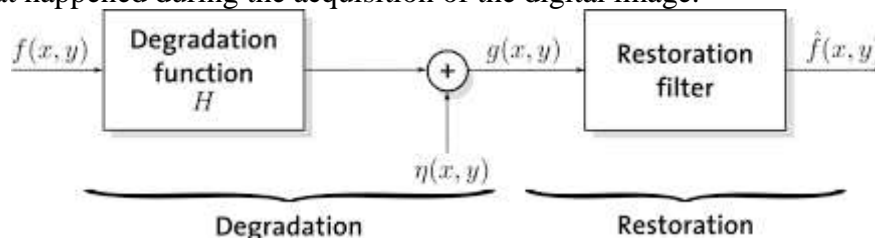


**Fig.2** Image Enhancement (image search: Digital Image Processing Tutorial)

Histogram equalisation is a widely used method to enhance the visual quality of images. Let's assume that we have a primarily dark picture. All the features of the image would then be squeezed into the dark end of the histogram, which would be skewed towards the lower end of the grey scale. The image will be clearer if we can extend the grey levels at the dark end to create a more evenly dispersed histogram.

### Image Restoration Methods

Through the removal or reduction of noise, blur, or other artefacts, degraded images can be restored using image restoration techniques. These techniques frequently entail estimating the original image from the degraded version using mathematical models. Restoration enhances the image in a certain way. It is a methodical procedure. Restoration makes use of past knowledge about the degradation phenomenon to attempt to rebuild a degraded image. These methods focus on modeling the degradation and recovering the original image by applying the inverse procedure. Image processing models, whether mathematical or probabilistic, serve as the foundation for restoration procedures.<sup>14</sup> Human subjective choices about what makes an improvement result "good" form the basis for enhancement. A set of techniques known as "image restoration" aims to eliminate or minimize any deterioration that happened during the acquisition of the digital image.



**Fig.3** Image Restoration and Image Degradation Model (image src: BuzzTech)

There are various degradation models for image restoration

**Noise Reduction:** Removing unwanted noise that obscures image details, often modeled as additive or multiplicative noise.

**Blur Removal:** Correcting blurring caused by factors such as motion, defocus, or atmospheric conditions, modeled as convolution with a blur kernel.

**Compression Artifact Removal:** Mitigating artifacts introduced during image compression, such as blackness or ringing.<sup>15</sup>

### Image Segmentation Algorithms

Using parameters like pixel intensity or colour similarity, image segmentation algorithms divide an image into several segments or areas. Numerous applications, including object identification and medical picture analysis, make use of these techniques. Edge detection is a prominent tool in image processing and computer vision, specifically in the areas of feature extraction and detection, which seek to locate locations in a digital image where there are formal discontinuities or sharp changes in brightness.

### Image compression

The demand for efficient and standardized image compression algorithms has grown due to the quick expansion of digital imaging applications, such as high-definition television, multimedia, teleconferencing, and desktop publishing. The practice of lowering the amount of data needed to convey a specific amount of information is known as data compression.

### Mathematics applied in Image Processing

Here explain basic mathematics are applied in image processing

### Fourier Transform

Fourier transform is used to split an image into its sine and cosine components.<sup>16</sup> The input image is its spatial domain equivalent, while the transformation output is in Fourier or frequency domain representation. The DFT is a sampled Fourier transform and hence it contains only a collection of samples big enough to fully define the spatial domain image. It does not contain all the frequencies that make up an image. Since the picture in the spatial and Fourier domains is of same size, the number of frequencies and the number of pixels coincide in spatial domain image.

For a square image of size  $N \times N$ , the two-dimensional DFT is given by:

$$F(k, l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i, j) e^{-i2\pi\left(\frac{ki}{N} + \frac{lj}{N}\right)}$$

where  $f(a, b)$  is the image in the spatial domain and the exponential term is the basis function corresponding to each point  $F(k, l)$  in the Fourier space. The basic functions are sine and cosine waves with increasing frequencies, i.e.  $F(0,0)$  represents the DC-component of the image corresponding to the average brightness and  $F(N-1, N-1)$  represents the highest frequency. The image in the Fourier domain can be re-transformed into spatial domain. The required inverse Fourier transform may be given by:

$$f(a, b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k, l) e^{i2\pi\left(\frac{ka}{N} + \frac{lb}{N}\right)}$$

Note the normalization term  $\frac{1}{N^2}$  in the inverse transformation.

A double sum is to be calculated for every image point, to solve the above equations. Nonetheless, the Fourier transform may be written as follows since it is separable:

$$F(k, l) = \frac{1}{N} \sum_{b=0}^{N-1} P(k, b) e^{-i2\pi\frac{lb}{N}}$$

Where

$$P(k, b) = \frac{1}{N} \sum_{a=0}^{N-1} f(a, b) e^{-i2\pi\frac{ka}{N}}$$

The spatial domain image is initially converted into an intermediate image through  $N$  one-dimensional Fourier Transforms with the help of these two formulas. The resulting intermediate image is later transformed into the final image, again with the help of  $N$  one-dimensional Fourier

Transforms. The number of computations can be decreased by expressing the two-dimensional Fourier Transform in terms of a series of  $2N$  one-dimensional transforms.

One of the most traditional tools in image processing is the two-dimensional Fourier transform. It is basically a complex exponential decomposition of a signal and an extension of the widely known Fourier transform for signals. Since pixels in image processing have actual intensities, complex oscillations always occur in pairs.

### **Linear algebra in Image Processing**

Matrices are used to represent images in image processing, where each pixel value in the image is corresponding to a matrix element. To solve the problems of convolution and filtering, these linear algebraic operations such as matrix multiplication, addition, and subtraction are performed on these matrices.<sup>17</sup>

Numerous applications, including image or signal processing, optimization etc use matrix operations such as matrix multiplication, matrix addition, and matrix inversion. The following equation represents the mathematical expression for homogeneity of a linear operator which may be applied to a picture in image processing:

$$Y(kf(x,y)) = kT(f(x,y))$$

Scaling and normalisation processes in image processing depend on this homogeneity, which is a basic characteristic of linear operators.

### **Additive**

$$g(x,y) = f(x,y) + c$$

Where is the  $g(x,y)$  output picture,  $f(x,y)$  is the input picture and  $c$  is the constant value added to each pixel in the input picture.

Images are frequently adjusted for brightness and contrast using additive linear operators. We can change an image's overall brightness or blackness by changing the value of  $c$ . The brightness will grow with a positive value of  $c$  and decrease with a negative value of  $c$ .

### **Averaging the images**

Averaging is one of the popular linear operators used in image processing. It entails calculating the average value of a set of pixels in a picture and substituting the average value for the set of pixels. This can be helpful for blurring or smoothing out an image, as well as for lowering noise in it.

### **Subtraction**

A basic operation in image processing is image subtraction, which is taking the pixel values belonging to two or more images and subtracting them to create a new image. Final image shows how the two original photographs differed from one another. Among the many uses for picture subtraction are object and motion detection as well as image enhancement.

Subtracting a reference image from the current image is called image subtraction. It is frequently used in object identification to separate foreground objects from the background. The difference image that is produced will show the locations in which the objects have changed, making it possible to identify and follow moving objects.

Image subtraction, which involves a subtraction of a background image from a current image, is used in motion detection to pinpoint the moving elements of a scene. The difference image that is produced will show the locations of the motion, making it possible to identify and follow moving objects or scene changes.

Subtraction of a filtered version of the image from the original one is used in image enhancement by eliminating noise and artifacts from the image. The difference image that is produced will show the regions of the image that include the artifacts and noise, making it possible to remove them using different image processing methods.

### Mathematical Morphology

One technique that enhances the image by producing better imaging and concentrating on the image's interest information is mathematical morphological image processing. For the objectives of observation and comparison, mathematical morphology is applied to the image utilising different structural element (SE) features. MATLAB software is used in mathematical morphology.<sup>18</sup>

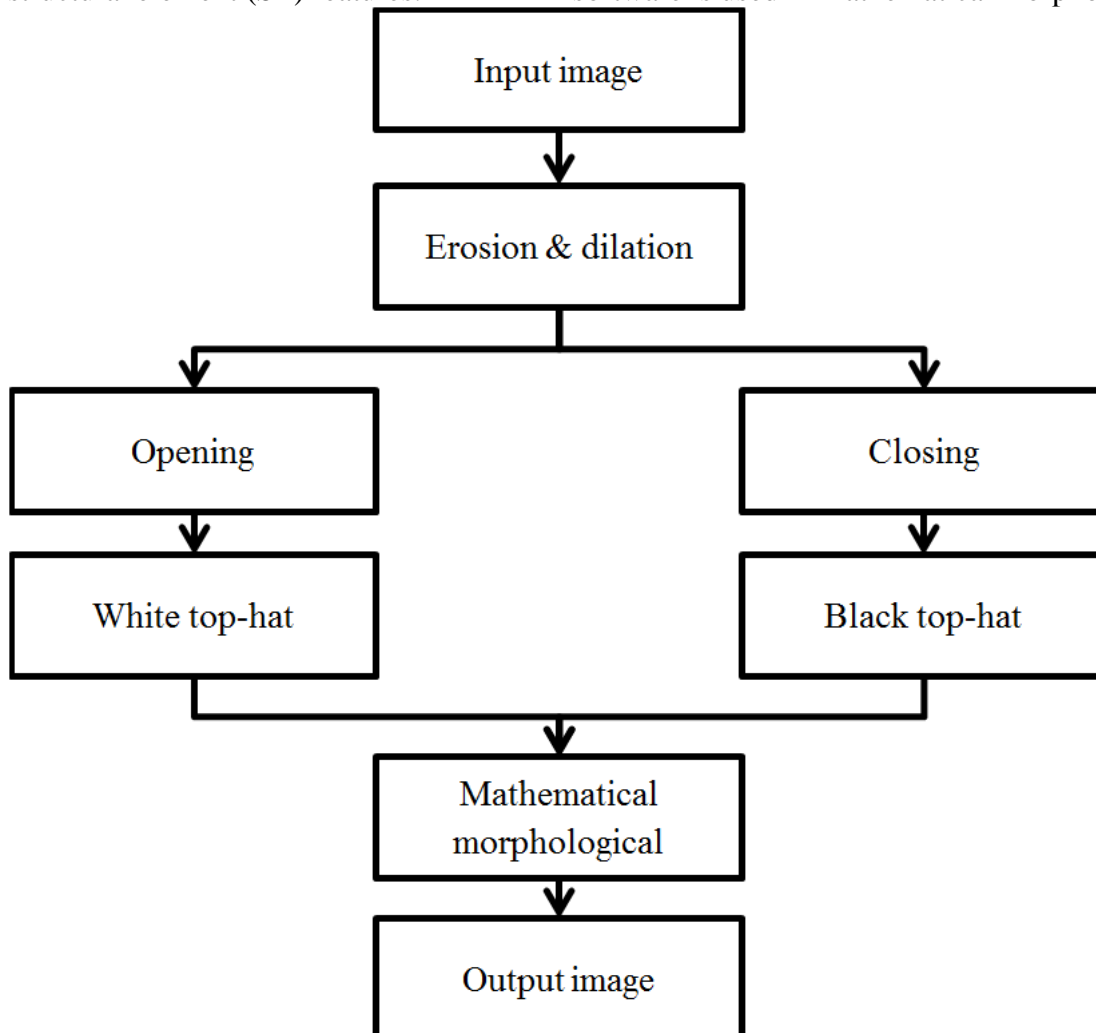


Fig. 4. Flow chart showing process of mathematical morphological image processing. (Khairul Anuar Mat Said, 2016)<sup>19</sup>

The white top-hat (WTH) and black top-hat (BTH) transformations are the two main operations of the mathematical morphological processes. WTH uses the differences in results obtained between the input image (IM) and the opening operation to improve the image's foreground structures, whereas BTH uses the differences in results between the IM and the closing operation to improve the image's background structures. The following is a definition of the mathematical morphological WTH and BTH transformations:

$$\text{Mathematical morphological} = \text{IM} + \text{WTH} - \text{BTH}$$

$$\text{WTH} = \text{IM} - \text{Opening}$$

$$\text{BTH} = \text{Closing} - \text{IM}$$

### Mathematical Methods in Image Processing

Table 4 provides an explanation of the various mathematical techniques and instruments utilized in image processing.

Table 4. Different types of mathematical methods used in IP

Mathematical Method	Application in Image Processing
Histogram Equalization	<p>A more advanced method is called histogram equalization, which modifies an image's dynamic range by changing the pixel values in response to the image's intensity histogram. A flat (or nearly flat) histogram is the outcome of histogram equalisation, which produces a nonlinear mapping by reassigning the intensity values in the input image to produce images with a uniform distribution of intensities. This mapping process makes use of a lookup table. Because it utilises the dynamic range more effectively, the final image usually highlights more image details [20]. The following are the steps involved in histogram equalization:</p> <ol style="list-style-type: none"> <li>1. Determine the input image's histogram.</li> <li>2. Adjust the generated histogram's normal to the interval [0, 1].</li> <li>3. Place the histogram that has been normalised into a colour table.</li> <li>4. Use the lookup table to pass the input image through.</li> </ol> <p>Histogram equalisation is a popular statistical approach that is used to enhance visual contrast. <math>f(x, y)</math> is the input image, which can be either a light, dark, or low contrast image. An image with strong contrast, <math>g(x, y)</math>, is the result.<sup>20</sup></p>
Spatial Linear Filters	<p>A spatial filter is made up of a pixel's specific neighbourhood and a predetermined procedure that is applied more to the picture pixels that the neighborhood contains. There are two types of spatial linear filters.</p> <ol style="list-style-type: none"> <li>(i) linear spatial filters are smoothed (they are smoothed models)</li> <li>(ii) refining spatial filters (they are linear models)</li> </ol> <p>Local averaging, also referred to as blurring, is a known side effect of smoothing that is comparable to spatial summation or spatial integration. Sharp edges become blurry during the smoothing process, but little details and noise are lost. Operations that sharpen and smooth provide opposite results. A blurry image <math>f</math> can be sharpened by using smoothing, which also enhances the image's edge.</p>
Discrete cosine Transformations	<p>In the field of signal processing, one of the most effective methods is the discrete cosine transform (DCT). The Fast Fourier Transform (FFT) is a mathematical operation approach used in DCT execution. FFT converts a representation from one type to another after receiving a single signal as input. A signal in the spatial domain is converted into a signal in frequency domain in this translation procedure. Redundancy of information is minimized throughout this process because the kernel functions—cosines—compose an orthogonal foundation. The principal advantage of the DCT transforms is that they are easier to apply and simultaneously they provide great energy compactness. This results in DCT coefficients being able to completely define the signal which is being analysed.</p>
Laplacian Distribution	<p>One of the methods most frequently used to analyse images is the Laplacian distribution. There are known problems with the Laplacian approach's incapacity to accurately describe image edges since it makes use of invariant Gaussian kernels. For edge-centric procedures like tone mapping, smoothing, and image preservation, this method is therefore not advised.</p>

**Mathematical Tools**

Table 5. Different types of mathematical tools are used in image processing<sup>21</sup>

<b>Mathematical Tools</b>	<b>Application in Image Processing</b>
MATLAB	One of Math Works' packages is MATLAB. One of the popular tools used for creating models with AI technologies is this one, which works with fuzzy, ANN, and hybrid algorithms like ANFIS. Image Processing Toolbox is software included with MATLAB. There are many different mathematical features and algorithms available in this toolkit. Pictures can be thought of as matrices with the image's pixel values as their elements. MATLAB's matrix functions enable the examination of picture characteristics. The program does not distinguish between binary and greyscale images; it treats 24-bit RGB images similarly to greyscale images. Additionally, MATLAB makes it simple to convert images between different image types. <sup>22</sup>
Scilab Image Processing (SIP) toolbox	Another popular open-source program in the field of image processing is SIP. This tool facilitates the fast prototyping of imaging solutions and can carry out a variety of tasks, including processing MRI pictures to provide a prompt diagnostic and image segmentation, filtering, edge detection, and other tasks.
GNU Octave	Numerical computing is the primary application of GNU Octave. The tool solves numerous numerical, linear, and non-linear problems using a high-level programming language and the Command Line Interface (CLI). With pre-packaged picture plotting and visualisation tools, the programming language supports picture processing activities. <sup>23</sup>
Sage Math	SageMath is a mathematically-based image processing tool that is constructed from multiple open-source tools. It can also generate animated plots in addition to 2-D and 3-D graphics and photos. It is used to create applications for image processing that identify the letter contained in an ornamental letter image, enhance the way a panorama is visualised perceptually, and apply recovery techniques to images in order to solve a set of linear Boolean equations. <sup>24</sup>

**Artificial Intelligence**

Visualisation, picture repair and sharpening, image retrieval, object measurement, and pattern detection are the main goals of image processing. Depending on the type of image, RGB processing or pseudocolour processing is used in colour image processing.

By utilising AI platforms to carry out difficult operations like text recognition, face recognition, and object identification. The process of filtering improves and modifies the supplied image. The most recent filtering methods include wiener, median, and linear filtering.<sup>25</sup>

**Machine learning**

The following table 6 explains different Machine learning frame works in IP

**Table 6. Applications of Machine learning in IP**

<b>M L frame works</b>	<b>Application in IP</b>
Tensor flow	It is open source model and includes a set of libraries for image processing projects. <sup>26</sup>
Pytorch	This frame work has Python, C++ and Java interface techniques used n IP
MATLAB(Matrix Lab)	This platform offers a toolset for image processing that contains several methods and work flow applications for processing, analyzing, and visualizing images using artificial intelligence. 3D image processing, picture segmentation, noise reduction, and image enhancement may all be

	done with Matlab IPT. <sup>27</sup>
Microsoft Computer Vision	Microsoft offers a cloud-based service called computer vision that allows you to access sophisticated algorithms for data extraction and intellectual property. It enables text extraction from photos, content moderation, and visual feature analysis.
Google Cloud Vision	This model offers a variety of image processing features like object localization, object recognition, image labelling and classification etc.
Google colab	This platform enables AI developers to increase their knowledge of image processing through better coding skills.
Neural Networks	The most useful tools for IP are found on this platform, and computational models—which consist of interconnected artificial neurones—can be used to solve a variety of image processing issues. <sup>28</sup>
Convolution Neural Network (CNN)	CNNs transform the field of image analysis. Comprising convolutional, pooling, and fully connected layers, they are intended for input data that has a structure like a grid. Filters, also called as kernels, are used for extraction of features at different spatial places, which allows convolutional layers to extract regional patterns like edges, corners, and textures. <sup>29</sup>
Mask R-CNN	A convolutional neural network (CNN) is a tool used by deep learning models like Mask R-CNN for identification of objects and segmentation of instances in photos. It is employed in segmentation tasks such as organ delineation and tumor identification.
Fully CNN	Convolutional neural networks, also called as ConvNets or CNNs, belong to deep learning network architectures. They get their working knowledge directly from the data. They are very useful in identifying patterns in photos to define objects, classes, and categories. Additionally, they may be combined to effectively categorise signal, time-series, and audio data. <sup>30</sup>
U-Net	A convolutional neural network called U-Net was created for the purpose of segmenting images. It is built on a fully convolutional neural network, whose architecture has been modified to produce more accurate segmentation using less number of training photos. Using the U-Net architecture applications of U-Net in biomedical image segmentation, like brain image segmentation ("BRATS"), liver image segmentation ("siliver07"), protein binding site prediction etc, the segmentation of a 512 × 512 image takes less than a second on a modern (2015) GPU. <sup>31</sup>
GAN	They consist of a generator network and a discriminator network that can learn to produce realistic images while differentiating between artificial and real visuals. During training, the generator and discriminator networks compete with each other to produce indistinguishable images, while the discriminator aims to correctly identify photos. <sup>32</sup>

### Image processing in various applications

The following table 7 shows the applications of Image processing in various domains

**Table 7. Different fields are applied in IP**

<b>Image processing in various fields</b>	<b>Applications</b>
Medical Image Processing	In medical image processing, mathematical techniques are essential because they allow medical images to be analysed and interpreted for both therapeutic and diagnostic purposes. Medical pictures, including X-rays, MRIs, and CT scans, can have information extracted from them using techniques including image

	segmentation, registration, and classification. <sup>33</sup>
Satellite Image Analysis	Processing and interpreting photos taken by satellites for use in urban planning, disaster relief, and environmental monitoring is known as satellite image analysis. Useful information is extracted from satellite images using mathematical techniques such as object recognition, change detection, and image categorization. <sup>34,35</sup>
Forensic Image Analysis	In criminal investigations, forensic image analysis is used to examine and improve photos connected to illegal activity. Information such as fingerprints, footprints, and facial features can be extracted from photos using mathematical techniques including image enhancement, pattern recognition, and image comparison. <sup>36</sup>
Industrial Image Processing	Images are inspected and analysed using industrial image processing in the manufacturing and quality control processes. To guarantee product quality and process efficiency, mathematical techniques including picture segmentation, pattern recognition, and defect detection are applied. <sup>37,38</sup>

### Cloud computing for image processing

Cloud computing transforms data processing and storage through the provision of scalable and on-demand access to computational resources via the internet. It provides a flexible and reasonably priced solution for image processing applications requiring a lot of processing power and storage capacity. Because the cloud architecture offers a wide range of services, including virtual machines, storage, and specialized tools, and is delegated to remote servers, it is suitable for a variety of image processing applications.<sup>39</sup>

### Cloud-Based Image Processing Platforms

Cloud-based image processing platforms simplify the development and deployment of applications by providing tools and services for image analysis and image modification through pre-built algorithms, libraries, and frameworks. Among the well-known platforms are AWS, Microsoft Azure, and Google Cloud Platform. These platforms enable developers to integrate image processing capabilities into their apps and upload, process, and store large amounts of photographs rapidly with less infrastructure setup and maintenance by providing APIs and SDKs.<sup>40</sup>

### Conclusion

Application of mathematics plays a key role in image processing. It provides the theoretical foundations and computational techniques and tools necessary for analyzing, manipulating, and interpreting digital images. From digital image representation to recent techniques such as spatial filtering, image enhancement, and pattern recognition, mathematics forms the basis of every aspect of image processing. By utilizing mathematical principles and algorithms, researchers and engineers can develop efficient and effective practical solutions for a wide range of applications, spanning from medical imaging to satellite imagery analysis. With technical advancement the integration of mathematics and image processing will drive further innovation, enabling new discoveries and applications across various domains. The Artificial intelligence and machine learning platforms also help to improve the image processing. At present situation the cloud computing also enhances the image processing techniques.

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