

Geometry in Vedic Tradition and Other

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Abstract

The important branch of mathematics which received earliest attention was Geometry and it is well explained in the texts of ancient cultures. In this paper I made an attempt to explore how geometry was developed and also discussed various results obtained by Vedic scholars. The beginning of geometry can be traced to ancient Mesopotamia, Egypt, Babylonian and India. We find many geometrical patterns in nature also Pythagoras was probably one of the first to give a deductive proof of Pythagoras theorem. Thales expanded the range of geometry. Geometry flourished in India, Arabia, China and Europe in 9th century. Analytic geometry, projective geometry, non-Euclidean geometry and manifolds were developed in 17th century. There are eight Sulva Sutras, most notable were the Baudhayana, Apasthamba and Katyayana. In the construction of Mahavedi and altars properties of right angle triangle were used. The epithet is Ksetra Ganita, Rekha Ganita and Ksetramiti. In the construction of Mahavedi and altars properties of right angle triangle were used. M. Cantor and others recognize that Pythagoras theorem was known to Indians before 8th century BC.

Keywords – Ksetra Ganita, Pythagorean, Sulva Sutras and Vedas

Introduction

Geometry is a branch of mathematics concerned with shape, size, relative position of figures, and the properties of space. The earliest recorded beginnings of geometry can be traced to ancient Mesopotamia and Egypt in the 2nd millennium BC [11]. By the 3rd century BC, geometry was put into an axiomatic form by Euclid, whose treatment, Euclid's Elements set a standard for many centuries to follow [14].

Greek expanded the range of geometry to many new kinds of figures, curves, surfaces, and solids; they changed its methodology from trial and error to logical deduction. Geometry began to see elements of formal mathematical science emerging in the west as early as the 6th century BC [2]. Euclidean geometry includes the study of points, lines, planes, angles, triangles, congruence, similarity, solid figures, circles, and analytic geometry [19]. Topology is the field

concerned with the properties of geometric objects that are unchanged by continuous mappings. Convex geometry investigates convex shapes in the Euclidean space and its more abstract analogues, Algebraic geometry studies geometry through the use of multivariate polynomials and other algebraic techniques. Discrete geometry is concerned mainly with questions of relative position of simple geometric objects, such as points, lines and circles.

Geometrical Pattern found in Nature - Living things like orchids, hummingbirds, and the peacock's tail have abstract designs with a pattern and color that artists struggle to match [5]. Mathematics seeks to discover and explain abstract patterns or regularities of all kinds [20]. Symmetry is universal in living things. Animals mainly have bilateral or mirror symmetry, as do the leaves of plants and some flowers such as orchids [21]. Plants often have radial or rotational symmetry, as do many flowers and some groups of animals such as sea anemones. Five- fold symmetry is found in the echinoderms, the group that includes starfish, sea urchins, and sea lilies.

Example:

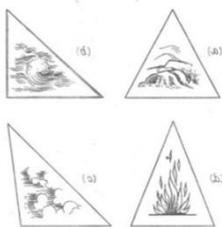
Starfish (Fivefold symmetry)



Cycas Circinalis (Rotational Symmetry)



Triangles from Nature



Hexagon from Nature



Use of Geometry in daily life since early age The earliest known unambiguous examples of



written records dating from Egypt and Mesopotamia about 3100 BCE, demonstrate that ancient peoples had already begun to devise mathematical rules and techniques useful for surveying land areas, constructing buildings, and measuring storage containers. The earliest recorded beginnings of geometry can be traced to early peoples, in the ancient Indus Valley civilization, and ancient Babylonian civilization from around 3000 BC. There were some surprisingly sophisticated principles, and it might be hard to put to derive some of them without the use of calculus ; the Egyptians had a correct formula for the volume of a frustum of a square pyramid.

Indus Valley Civilization

Geometry in different countries since ancient time –

Babylonian geometry - There have been recent discoveries showing that ancient Babylonians might have discovered geometry nearly 1400 years before Europeans did. The Pythagorean Theorem was also known to the Babylonians.

Thales (635 - 543 BC) of Miletus (now in southwestern Turkey), used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. He is credited with the first use of deductive reasoning applied to geometry, by deriving four corollaries to Thales' Theorem [2]. Thales strongly believed that reasoning should supersede experimentation and intuition, and began to look for solid principles upon which he could build theorems. This introduced the idea of proof into geometry and he proposed some axioms that he believed to be mathematical true.

Pythagoras (582-496 BC), of Ionia, and later, Italy, then colonized by Greeks, may have been a student of Thales. The theorem that bears his name may not have been his discovery, but he was probably one of the first to give a deductive proof of it.

Hippocrates took the development of geometry further. He was the first to start using geometrical techniques in other areas of mathematics. He studied the problem of squaring the circle which is not perfect, simply because Pi (π) is an irrational number.

Apollonius of Perga (262 - 190 BCE) - He was a mathematician and astronomer, and he wrote a treatise called 'Conic Sections'. He is credited with inventing the words ellipse, parabola, and hyperbola, and is often referred to as the great Geometer.

Egypt Geometry –

Euclid was associated with the cosmopolitan University of Alexandria, Euclid may well have been an Egyptian or a Jew but like others of the school he wrote in Greek his thirteen books composed about 300 BC, Euclid himself wrote eight more advanced books on geometry [13]. He was brought to the university at Alexandria by Ptolemy I, King of Egypt. Around 300 BC, geometry was revolutionized by Euclid, whose *Elements*, widely considered the most successful and influential textbook of all time [1], introduced the axiomatic method and is the earliest example of the format still used in mathematics today, that of definition, axiom, theorem, and proof. Euclid arranged them into a single, coherent logical framework [10]. The **Elements** were known to all educated people in the west until the middle of the 20th century and its contents are still taught in geometry classes today [8].



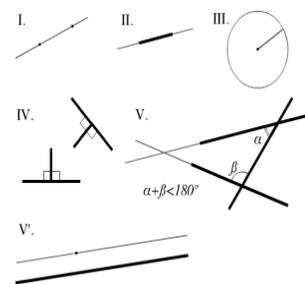
Euclid

Points - In many areas of geometry, such as analytic geometry, differential geometry, and topology, all objects are considered to be built up from points [6].

Line- In analytic geometry, a line in the plane is often defined as the set of points whose coordinates satisfy a given linear equation [12]. For instance, planes can be studied as a topological surface without reference to distances or angles [16].

Following are five axioms of Euclid.

1. Any two points can be joined by a straight line.
2. Any finite straight line can be extended in a straight line.
3. A circle can be drawn with any center and any radius.
4. All right angles are equal to each other.
5. If two straight lines in a plane are crossed by another straight line called the transversal, and the interior angles between the two lines and the transversal lying on one side of the transversal add up to less than two right angles, then on that side of the transversal, the two lines extended will intersect (also called the parallel postulate).



Euclid's fifth postulate cannot be proven as a theorem. Euclid himself used only the first four postulates, but was forced to invoke the parallel postulate.

Archimedes (287-212 BC), of Syracuse, is often considered to be the greatest of the Greek mathematicians; he developed methods very similar to the coordinate systems of analytic geometry. Geometry was connected to the divine for most medieval scholars. The compass in this 13th-century manuscript is a symbol of God's act of Creation.

Arabia -In the Middle Ages, mathematics in medieval Islam contributed to the development of geometry, especially algebraic geometry [18]. Three scientists, Ibn al-Haytham, Khayyam, and al-Tusi, had made the most considerable contribution to this branch of geometry whose importance came to be completely recognized only in the 19th century. The theorem on quadrilaterals, including the Lambert quadrilateral in which three of its angles are right angles works had a considerable influence on the development of non-Euclidean geometry.

China -The Chinese knew the relation $3^2 + 4^2 = 5^2$ in the time of Chou Kong (1105 BC). The first definitive work on geometry in China was the Mo Jing [15], the Mohist canon of the early philosopher Mozi (470-390 BC). It was compiled after his death by his followers around the year 330 BC [17]. However, due to the infamous Burning of the Books in a political maneuver by the Qin Dynasty ruler Qin Shihuang (221-210 BC), multitudes of written literature created before his time was purged. This book included many problems where geometry was applied and included the use of the Pythagorean Theorem. The book provided illustrated proof for the Pythagorean Theorem [9].

India- Geometry arose independently in India, with texts providing rules for geometric constructions appearing as early as the 3rd century BC, in Vedic .

Geometry in Vedic Culture –Indian mathematicians also made many important contributions in geometry. The Satapatha Brahmana (3rd C. BC) contains rules for ritual geometric constructions that are similar to the Sulba Sutras. According to Hayashi, the Sulba Sutras contain "the earliest extant verbal expression of the Pythagorean Theorem", although it had already been known to the old Babylonians. In the Bakshali manuscript, there are a handful of geometric problems. Aryabhata (499 AD) includes the computation of areas and volumes, he stated his theorem on the diagonals of a cyclic quadrilateral and complete description of rational triangles [7].

The Sulba Sutras in the Vedas is a manual of geometrical constructions [22]. The Taittiriya Samhita of Yajurveda gives the measurement of **Mahavedi** with a right angle of sides 15, 36

and hypotenuse 39 units. Katyayana gives construction of a right angle triangle with Sides $\frac{n^2 - 1}{2}$ a, n a, and a hypotenuse of length $\frac{n^2 + 1}{2}$ a. Such construction was used in building the Vedis using the properties of similar triangles. It is surprising to find that an instrument was actually used for drawing circles in the Indus valley as early as 2500 BC. The date of oldest Sulva Sutra is said to be 8th century BC [4]. **Baudhayana and Apasthamba** list several right angle triangles of different measurement (triplets) [23].

Application- Geometry has applications in many areas, including the art of writing or solving codes and in string theory (string theory is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings). Discrete geometry is concerned mainly with questions of relative position of simple geometric objects, such as points, lines and circles. Euclidean geometry also has applications in computer science, crystallography (crystallography is a technique used for determining the atomic and molecular structure of a crystal), and various branches of modern mathematics. An important area of application is number theory. In ancient Greece the Pythagoreans considered the role of numbers in geometry. Since the 19th century, geometry has been used for solving problems in number theory, for example through the geometry of numbers or, more recently, scheme theory, which is used in Wiles's proof of Fermat's Last Theorem.

Conclusion- We found that geometry is well explained in all the philosophical and mathematical texts of different cultures. Mathematicians and others developed geometry for different purposes. There was a pervasive fascination with geometrical results. It will motivate further studies and research of ancient and medieval geometry. We have shown that geometry grew independently in different culture. Indians had also good knowledge of geometrical calculations and their approach was scientific. We should not forget that all these accomplishments were made in the absence of the modern mathematical techniques. Indian ancient texts remained unexposed to the western countries due to several reasons and the history was written by English so no importance was given to Indian mathematicians by foreigners. [3] Certainly, it seems that Indian contributions to geometry has not been given due acknowledgement until very recently in modern history of mathematics.

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