FISHER-TYPE FIXED POINT RESULTS in B-METRIC SPACES for FOUR MAPPINGS

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Abstract

In this paper is to clearly formulate various possible assumptions for a comparison function in contractive conditions and prove some common fixed-point theorems for three self-mappings in the context of a complete b-metric space by proposing a new contractive type condition. Further, we derive a result for four self-mappings in the same setting.
Keywords: b-metric space; common fixed point; weakly compatible.

Introduction

Fixed-point theory was a results of the investigation of the existence and uniqueness of a solution of certain differential equations. In 1922, Banach [9] reported an elegant fixed-point theorem .In 1993, Czerwik [17] suggested a successful and proper generalization of the metric space notion by introducing the concept of b-metric space. Following this famous result in the setting of b-metric spaces, several extensions in distinct aspects have been released in this direction (see e.g., [2-4,6-7,12-15] and references therein).In this paper, we study certain common fixed-point theorems for four maps in the setting of complete b-metric spaces. Firstly, we recall the notion of b-metric.

Preliminaries

Definition 1 [17]. Let X be a non empty set. A function $d : X\times X \to [0,\infty)$ is called a b-metric if the following axioms are fulfilled:

(b1) d is reflexive, that is, $d(x,y) = 0$ if and only if $x = y$.
(b2) d has a symmetry, that is, $d(x,y) = d(y,x)$ for all $x,y\in X$.
(b3) $d(x,y) \leq s[d(x,z) + d(z,y)]$ for all $x,y,z\in X$, where $s\geq 1$.
Here, (X,d) is called a b-metric space.

Remark 1. In case of s = 1,the b-metric coincide the standard metric. Notice that b-metric does not need to be continuous in general. In this manuscript, we deal with continuous b-metrics only [11].
Example of b-metric.
Example 1. Let $X = \{x_1 : 1\leq i \leq M\}$ for some $M \in N$ and $s\geq 2$. Define $d : X\times X \to\infty$ as
$d(x_i,x_j) = 0$ if $i = j$,

= s if (i,j) = (1,2) or (i,j) = (2,1),

= 1 otherwise.

Consequently, we derive that

$d(x_i,x_j) \leq s/2\,[d(x_i,x_k) + d(x_k,x_j)]$, for all i,j,k ∈ {1,M}.

Thus, (X,d) forms a b-metric for s > 2 where the ordinary triangle inequality does not hold.

Example 2. (See e.g., [12]) For 0 < q < 1, the space $L^q[0,1]$ of all real-vauled functions f(t), t∈ [0,1] such that $\int_0^1 |f(t)|^q\,dt < \infty$, endowed with

$d(f,h) := \left(\int_0^1 |f(t)-h(t)|^q\,dt\right)^{1/q}$, for each x,y∈ $L^p[0,1]$, forms a b-metric space. Notice that $s = 2^{1/q}$.

Definition 2. (see e.g., [1,20]) Suppose that f and g are self mappings on a non-empty set X. A point x is names as a coincidence point of f and g in case fx = gx, for x in X. Moreover, z is called a point of coincidence of f and g whenever z = fx = gx for some x in X. In addition, f and g are said to be weakly compatible, if fx = gx ⇒ f(gx) = g(fx) holds for every x ∈ X.

Proposition 1. (see Lemma 3 in [5]) Let f,g,h be self mappings on a non-empty set X and v ∈ X is the a unique coincidence point of f,g and h. These self-mappings, f,g,h, have a unique common fixed point if{f,h} and{g,h}are weakly compatible.

Definition 3. [20,21] A function φ : [0,∞) → [0,∞) is called a comparison function if it is increasing and $\varphi^n(t) \to 0$ as n→∞ for every t∈ [0,∞), where φ^n is the n-th iterate of φ.

Lemma 1. ([20,21]) If φ : [0,∞) → [0,∞) is a comparison function, then

1. each iterate φ^k of φ, k ≥1 is also a comparison function;

2. φ is continuous at 0;

3. φ(t) < t for all t > 0.

Definition 4. Let s ≥ 1be a real number. A function ψ : [0,∞) → [0,∞) is called a (b)-comparison function if

1. ψ is increasing;

2. there exist k_0 ∈ N, a ∈ (0,1) and a convergent nonnegative series $\sum_1^\infty v_k$ such that $s^{k+1}\psi^{k+1}(t) \leq a\, s^k \psi^k(t) + v_k$, for $k \geq k_0$ and any t≥0.

Let Ψ = {ψ : [0,∞) → [0,∞) : ψ is b−comparison function}. Note that in case of s = 1, a (b)-comparison function is named as (c)-comparison.

Lemma 2. ([11]) For φ ∈Ψ,

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1. the series $\sum_1^\infty s^k \varphi^k(t)$ converges for any $t \in [0,\infty)$;

2. the function $b_s : [0,\infty) \to [0,\infty)$ defined as $b_s = \sum_1^\infty s^k \varphi^k(t)$ is increasing and continuous at $t = 0$.

Remark 2. On account of Lemma 2 and Lemma 1, any (b)-comparison function, we have ψ satisfies $\psi(t) < t$.

Fisher [18] proved the following existence theorem:

Theorem 1. [18] Let T be a mapping of the complete metric space X into itself satisfying the inequality $[d(Tx,Ty)]^2 \leq a(d(x,Tx)d(y,Ty)) + b(d(x,Ty)d(y,Tx))\ \forall x,y \in X,\ 0 \leq a < 1,\ 0 \leq b$

then T has a fixed point in X.

In 1980, Pachpatte [23] extended the result of Fisher [18] in the following way.

Theorem 2. [23] Let T be a mapping of the complete metric space X into itself satisfying the inequality $[d(Tx,Ty)^2 \leq a[d(x,Tx)d(y,Ty) + d(x,Ty)d(y,Tx)] + b[d(x,Tx)d(y,Tx) + d(x,Ty)d(y,Ty))\ \forall x,y \in X$,

where $a,b \geq 0$ and $a+2b < 1$ then T has a unique fixed point in X.

[8] proved the following existence theorem:

Let (X,d) be a complete b-metric space and let f,g,h be mappings from X into itself satisfying the condition: $f(X) \cup g(X) \subseteq h(X)$. (i)

Let $x_0 \in X$. By (1) there exists a point $x_1 \in X$ such that $hx_1 = fx_0$ and for x_1 there exists $x_2 \in X$ such that $hx_2 = gx_1$. Inductively we can define the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$y_{2n} = hx_{2n+1} = fx_{2n},\ y_{2n+1} = hx_{2n+2} = gx_{2n+1}\ \forall n \geq 0$. (ii)

Lemma 3. Let f,g,h be mappings from a b-metric space (X,d) into itself satisfying (1) and such that for all $x,y \in X$ $d(fx,gy)]^2 \leq \psi(F(x,y))$, (iii)

where, $\psi \in \Psi$ and $F(x,y) = \max\{d(fx,gy)d(hx, fx), d(fx,gy)d(hy,gy), d(hy, fx)d(hx,gy), 1/2s\ d(hy,gy)d(hx,gy)\}$, $\psi \in \Psi$. Then, the sequence $\{yn\}$ defined by (2) is a Cauchy sequence in X.

Theorem 3. Let (X,d) be a complete b-metric space, f,g,h be self mappings of X satisfying the conditions (i) and(iii). We suppose also that h(X) is a closed subspace of X. Then the maps f,g and h have a coincidence point z in X. Moreover, if the pairs {f,h} and {g,h} are weakly compatible then f,g and h have a unique common fixed point in X.

Main Result

Let (X,d) be a b- metric space, and let $f,g,h,t : X \to X$.

Suppose that $f(X) \subset h(X),\ g(X) \subset t(X)$ (1)

and one of these four subsets of X is closed. Let further $d[fx , gy] \leq \phi F[x ,y]$

Let $x_0 \in X$. By (1) there exists a point $x_1 \in X$ such that $hx_1 = fx_0$ and for x_1 there exists $x_2 \in X$ such that $tx_2 = gx_1$. Inductively we can define the sequences {xn} and $\{y_n\}$ in X such that

$y_{2n} = fx_{2n} = hx_{2n+1}, y_{2n+1} = gx_{2n+1} = tx_{2n+2}, \forall n \geq 0.$ (2)

Lemma 4. Let f,g,h,t be mappings from a b-metric space (X,d) into itself satisfying (1) and such that for all $x,y \in X$

$$[d(fx,gy)]^2 \leq \psi(F(x,y)), \quad (3)$$

where, $\psi \in \Psi$ and

$F(x,y) = \max\{d(fx,gy)d(hx, fx),d(fx,gy)d(ty,gy),d(ty, fx)d(hx,gy),1/ 2sd(ty,gy)d(hx,gy)\},$

$\psi \in \Psi$. Then, the sequence {yn} defined by (2) is a Cauchy sequence in X.

Proof. For an arbitrary $x_0 \in X$, we shall construct a sequence {xn} and {yn} in (2). If there exists n_0 such that $Y\ 2n_0 = Y\ 2n_0+1$

we obtain : $hx_{2no+1}= fx_{2no} = tx_{2no+2} = gx_{2no+1}$

and hence, x_{2no+1} forms a common fixed point of h and g.

Without loss of generality, we suppose that $y_{2n} \neq y_{2n+1}$.

Accordingly, from(2)and(3)we find that

$$[d(y_{2n},y_{2n+1})]^2 = [d(fx_{2n},gx_{2n+1})]^2 \leq \psi(F(x_{2n},x_{2n+1})) \quad (4)$$

$F(x_{2n},x_{2n+1})=\max\{d(fx_{2n},gx_{2n+1})d(hx_{2n},fx_{2n}),d(fx_{2n},gx_{2n+1}),d(tx_{2n+1},gx_{2n+1}),d(tx_{2n+1},fx_{2n}),d(hx_{2n},gx_{2n+1}),1/ 2sd(hx_{2n+1},gx_{2n+1})d(hx_{2n},gx_{2n+1})\},$

$\leq\max\{d(hx_{2n+1},tx_{2n+2})d(hx_{2n},hx_{2n+1}),d(hx_{2n+1},tx_{2n+2})d(tx_{2n+1},tx_{2n+2}),d(tx_{2n+1},hx_{2n+1})d(hx_{2n},tx_{2n+2}),1/ 2sd(tx_{2n+1},tx_{2n+2})d(hx_{2n},tx_{2n+2})\},$

$\leq\max\{d(y_{2n},y_{2n+1})d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1})d(y_{2n},y_{2n+1}),d(y_{2n},y_{2n})d(y_{2n-1},y_{2n+1}),1/ 2sd(y_{2n},y_{2n+1})d(y_{2n-1},y_{2n+1})\},$

$\leq\max\{d(y_{2n},y_{2n+1})d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1})d(y_{2n},y_{2n+1}),d(y_{2n},y_{2n})d(y_{2n-1},y_{2n+1}),1/2s\ d(y_{2n},y_{2n+1})[d(y_{2n-1},y_{2n})+ d(y_{2n},y_{2n+1})\},$

Suppose $d(y_{2no-1},y_{2no}) < d(y_{2no},y_{2no+1})$ for some n_0.

Since the function then the inequality (4) turns into

$[d(y_{2no},y_{2no+1})]^2 \leq \psi([d(y_{2no},y_{2no+1})]^2) < [d(y_{2no},y_{2no+1})]^2$

which is a contradiction.

Thus, we have $d(y_{2n},y_{2n+1}) \leq d(y_{2n-1},y_{2n})$ for all $n \in N$.

Keeping in mind that ψ is non-decreasing, and by taking the inequality (4) into account and employing Remark 2 recursively, we conclude also that

$$[d(y_{2n},y_{2n+1})]^2 \leq \psi([d(y_{2n-1},y_{2n})]^2) < [d(y_{2n-1},y_{2n})]^2$$
$$\leq \psi^2([d(y_{2n-2},y_{2n-1})]^2) < [d(y_{2n-2},y_{2n-1})]^2]$$
$$\ldots\ldots\ldots\ldots$$
$$\leq \psi^{2n}([d(y_0,y_1)]^2).$$

By using the same arguments, similarly, we find that

$d(y_{2n-1},y_{2n}) \leq d(y_{2n-2},y_{2n-1})$,

and moreover,

$$[(y_{2n-1},y_{2n})]^2 \leq \psi([d(y_{2n-2},y_{2n-1})]^2) < [d(y_{2n-2},y_{2n-1})]^2$$
$$\leq \psi^2([d(y_{2n-3},y_{2n-2})]^2) < [d(y_{2n-3},y_{2n-2})]^2$$
$$\ldots$$
$$\leq \psi^{2n-1}([d(y_0,y_1)]^2).$$

As a result, for all $n \in N$, we get $[d(y_n,y_{n+1})]^2 \leq \psi([d(y_{n-1},y_n)]^2) < [d(y_{n-1},y_n)]^2 \leq \cdots < \psi^n([d(y_0,y_1)]^2)$. (5)
On the account of Lemma 2, we conclude that

$\lim_{n\to\infty} d(y_{n+1},y_n) = 0.$ (6)

Now, we shall indicate that the sequence {yn} is Cauchy.

By using the modified triangle inequality (b3) recursively, and keeping the fact that $(\alpha + \beta)^2 \leq 2(\alpha^2 + \beta^2)$ in mind, we observe the following estimation for the distance $d(y_n,y_{n+k})$ for $k \geq 1$ and $s \geq 1$

$$[d(y_n,y_{n+k})]^2 \leq [s(d(y_n,y_{n+1}) + d(y_{n+1},y_{n+k}))]^2$$
$$\leq 2s^2[d(y_n,y_{n+1})]^2 + 2s^2[d(y_{n+1},y_{n+k})]^2$$
$$\leq 2s^2[d(y_n,y_{n+1})]^2 + 2s^2\{s[d(y_{n+1},y_{n+2}) + d(y_{n+2},y_{n+k})]\}^2$$
$$\leq 2s^2[d(y_n,y_{n+1})]^2 + (2s^2)^2[d(y_{n+1},y_{n+2})]^2 + (2s^2)^2[d(y_{n+2},y_{n+k})]^2$$
$$\ldots$$

$\leq 2s^2[d(y_n,y_{n+1})]^2+(2s^2)^2[d(y_{n+1},y_{n+2})]^2+\cdots+(2s^2)^k[d(y_{n+k-1},y_{n+k})]^2$ (7)

Applying (5) and (7) we derive that

$[d(y_n,y_{n+k})]^2 \leq (2s^2)\psi^n([d(y_0,y_1)]^2) + (2s^2)^2\psi^{n+1}([d(y_0,y_1)]^2) +\cdots+ (2s^2)^k\psi^{n+k-1}([d(y_0,y_1)]^2)$

$= \frac{1}{(2s2)n-1}((2s^2)^n\psi^n([d(y_0,y_1)]^2) + (2s^2)^{n+1}\ \psi^{n+1}\ ([d(y_0,y_1)]^2) +\ldots +(2s^2)^{n+k-1}\psi^{n+k-1}([d(y_0,y_1)]^2).$

(8)

Consequently, we have $d^2(y_n,y_{n+k}) \leq 1/(2s^2)^{n-1}[P_{n+k-1}-P_{n-1}]$, $n\geq 1$, $k \geq 1$, (9)

where $P_n = \sum_{j=0}^{n} (2s^2)^j\psi^j([d(y_0,y_1)]^2)$, $n\geq 1$.

On the account of Lemma 2, the series

$\sum_{j=0}^{\infty} (2s^2)^j\psi^j([d(y_0,y_1)]^2)$ is convergent.

Since $s\geq 1$, letting limit $n\to\infty$ in (9) we deduce that

$\lim n\to\infty\ d^2(y_n,y_{n+k}) \leq \lim n\to\infty 1/(2s^2)^{n-1}[P_{n+k-1}-P_{n-1}] = 0.$ (10)

We find that the constructive sequence {yn} is Cauchy in (X,d).

Theorem 4. Let (X,d) be a complete b-metric space, f,g,h and t be self mappings of X satisfying the conditions (1) and(3). We suppose also that h(X) and t (X) is a closed subspaces of X. Then the maps f,g,h and t have a coincidence point z in X. Moreover, if the pairs {f,h}and{g,t} are weakly compatible then f,g,h and t have a unique common fixed point in X.

Proof. Let us consider now the sequence {yn} defined by (2). By Lemma 3, we have that {yn} is a Cauchy sequence in X and since X is complete, the sequence {yn} converges to a point z in X. But, h(X) is complete,being a closed subspace of X and since $f(X) \subseteq h(X)$ and $g(X) \subseteq t(X)$,the subsequences $\{y_{2n}\}$ and$\{y_{2n}\}$ which are contained in h(X) and t (X) must have a limit z in h(X) and t (X),

i.e. $\lim n\to\infty\ fx_{2n} = \lim n\to\infty\ gx_{2n+1} = \lim n\to\infty\ hx_{2n+1} = \lim n\to\infty\ tx_{2n+2} = z$.

Let $u\in h^{-1}z$ Then $hu = z$ and we suppose that $gu\neq z$.

From (3) we have

$[d(fx_{2n},gu)]^2 \leq \psi(F(x_{2n},u))$, (11)

where $F(x_{2n},u) = \max\{[d(fx_{2n},gu)d(hx_{2n}, fx_{2n})],[d(fx_{2n},gu)d(hu,gu)]\ [d(hu, fx_{2n})d(hx_{2n},gu)], 1/2s[d(hu,gu)d(hx_{2n},gu)]\}$.

Keeping Remark 2 in mind and by taking lim sup in (11) as $n\to\infty$,

we find that

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$[d(z,gu)]^2 \leq \psi([d(z,gu)]^2) < [d(z,gu)]^2$,

a contradiction.

Hence, we have $[d(z,gu)]^2 = 0$ which gives that gu = z = hu.

Using the similar reasoning, supposing that fu$\neq$ z

we have

$[d(fu,gx_{2n+1})]^2 \leq \psi(F(u,x_{2n+1}))$, (12)

where$F(u,x_{2n+1})=\max\{[d(fu,gx_{2n+1})d(hu,fu)],[d(fu,gx_{2n+1})d(hx_{2n+1},gx_{2n+1})],[d(hx_{2n+1}, fu)d(hu,gx_{2n+1})], 1/2s[d(hx_{2n+1},gx_{2n+1})d(hu,gx_{2n+1})]\}$.

Again, by taking Remark 2 into account and by letting lim sup in (12) as $n\rightarrow\infty$,

$[d(fu,z)]^2 \leq \psi([d(fu,z)]^2) < [d(fu,z)]^2$,

which is a contradiction.

Therefore, fu = z = hu = gu

Using the similar reasoning, supposing that tu$\neq$ z

From (3) we have

$[d(fx_{2n},tu)]^2 \leq \psi(F(x_{2n},u))$, (13)

where $F(x_{2n},u) = \max\{[d(fx_{2n},tu)d(hx_{2n}, fx_{2n})],[d(fx_{2n},tu)d(hu,tu)] [d(hu, fx_{2n})d(hx_{2n},tu)], 1/2s[d(hu,tu)d(hx_{2n},tu)]\}$.

Keeping Remark 2 in mind and by taking lim sup in (13) as $n\rightarrow\infty$,

we find that

$[d(z,tu)]^2 \leq \psi([d(z,tu)]^2) < [d(z,tu)]^2$,

a contradiction.

Therefore, fu = z = hu = gu =tu.

i.e., the maps f,g,h and t have a coincidence point. If we consider the supplementary assumption ,then the pairs(f,h) and (g,t),hare weakly compatible, we have

$hgu = ghu \Rightarrow gz = hz$

$hfu = fhu \Rightarrow fz = hz$,

tgh=gtu $\Rightarrow$gz = tz

thu=htu$\Rightarrow$ hz = tz

so t(z) = g(z) = f(z)=h(z). (14)

We shall show that z is the common fixed point of f,g ,h and t. Without loss of generality, suppose, on the contrary, that z$\neq$ gz. Hence, by (3) we get

$[d(fx_{2n},gz)]^2 \leq \psi(F(x_{2n},z)),$ (15)

where $F(x_{2n},z) = \max\{[d(fx_{2n},gz)d(hx_{2n}, fx_{2n})],[d(fx_{2n},gz)d(hz,gz)], [d(hz, fx_{2n})d(hx_{2n},gz)], 1/2s[d(hz,gz)d(hx_{2n},gz)]\}$.

By letting lim sup in (15) as n$\rightarrow\infty$, together with applying Remark 2, we find that

$[d(z,gz)]^2 \leq \psi([d(z,gz)]^2) < [d(z,gz)]^2$, a contradiction.

Thus, we have d(z,gz) = 0, that is, z = gz. By combining with (13) we get

fz = gz = hz = z which shows that z is a common fixed point of the mappings f,g and h. For the uniqueness,we suppose,on the contrary that f,g and h have two common fixed points z_1 and z_2 such that $z_1 \neq z_2$. Then, by using (3) we get

$[d(z_1,z_2)]^2 = [d(fz_1,gz_2)]^2\psi(F(fz_1,gz_2)),$ (16)

where

$F(fz_1,gz_2)=\max\{[d(fz_1,gz_2)d(hz_1,fz_1)],[d(fz_1,gz_2)d(hz_2,gz_2)][d(hz_2,fz_1)d(hz_1,gz_2)], 1/2s[d(hz_2,gz_2)d(hz_1,gz_2)]\}$

$\leq\max\{[d(z_1,z_2)d(z_1,z_1)],[d(z_1,z_2)d(z_2,z_2)] [d(z_2,z_1)d(z_1,z_2)], 1/ 2s[d(z_2,z_2)d(z_1,z_2)]\}$

$\leq [d(z_1,z_2)]^2$. Thus, (16) yields that

$[d(z_1,z_2)]^2 = [d(fz_1,gz_2)]^2\psi(F(fz_1,gz_2)) = \psi([d(z_1,z_2)]^2) < [d(z_1,z_2)]^2,$ (17)

a contradiction that completes the proof.

Conclusions

We prove some common fixed-point theorems for four self-mappings to use possible assumptions for a comparison function in contractive conditions.

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