STUDY OF IMPACT ON APPLYING ACCURACY VALIDATION OF SOME FIXED POINT THEOREMS FOR SEQUENCE OF MAPPINGS IN PROBABILISTIC METRIC SPACES

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ABSTRACT

The theory of metric spaces in several cases has applied to the association of a single number as the distance between a pair of elements is rather an over-idealization[3].Vasuki[4] generalized the result of Schweizer and Sklar[5] for a sequence of maps satisfying a new contraction type condition in Menger Spaces and proved a common fixed point theorem for a sequence of self maps[6].
Present paper is based on the basic theory and applications and their repercussion of probabilistic metric spaces (PM-space).[1] In this paper the descriptions of many probabilistically confined sets have been presented. In the form of repercussion obtained from the various sets, some definitions and theorem are presented to study the linear operator theory and fixed point theory on PM-spaces.[2]

KEYWORDS--*PM-spaces, Manger space, mappings, contraction pair, self-mappings, uniqueness*

INTRODUCTION

The purpose of this is to give some fixed point theorems for sequence of mappings in Menger spaces by generalizing the results of Vasuki,[7]

Definition 1.1 : Let (X, F) be PM space and $T_i : X \to X$, i = 1,2. two mappings. This pair of mappings is a contraction pair of (X, F) iff there is an $x \in (0, 1)$ such that

$$F_{T_1 q, T_2 p}(x) \geq F_{p,q}(x/x).$$

for every p, q $\in$ S.

Definition 1.2 : Let $\{T_i\}_{i=1,2}$ be a contraction pair and $p_0 \in S$. The sequence of iterates of p_0 under the pair $\{T_i\}_{i=1,2}$ is the sequence $\{p_n\}$ defined as follows :

$$p_{2j} = T_2(p_{2j-1})$$

$$p_{2i-1} = T_1(p_{2i})$$

For all i = 0,1,………. ; j = 1,2,……….

Theorem 1.3 : Let $\{T_n\}$ be a sequence of self-mappings of complete Menger space (X, F, t) into itself with t (u, v) = min (u, v) for every u, v $\in$ [0,1]. If for any two mappings T_1 and T_2, we have

(i) $F_{T_1^m u, T_2^m v}(\alpha p) \geq F_{u,v}(p)$.

(ii) $F_{T_1^m u, T_2 v}(\alpha p) \geq F_{u, T_2 v}(p)$

For some m and for u, v $\in$ X, $p > 0$ and $0 \leq \alpha < 1$, then for $x_0 \in X$ the sequence $\{x_n\}$ is defined as

$$x_n = T_n^m x_{n-1}, \quad n = 1,2,…………$$

converges and its limit is the unique common fixed point of T_n.

Proof : Let x_0 be any arbitrary point in X. Then for each $p > 0$ and $0 \leq \alpha < 1$,

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$$F_{x_1,x_2}(\alpha p) = F_{T_1^m x_0, T_2^m x_1}(\alpha p)$$

$$\geq F_{x_0,x_1}(p)$$

Again

$$F_{x_2,x_3}(\alpha p) = F_{T_1^m x_1, T_2^m x_2}(\alpha p)$$

$$\geq F_{x_1,x_2}(p)$$

and by induction

$$F_{x_{n+1},x_n}(\alpha p) \geq F_{x_n,x_{n-1}}(p)$$

For each $p > 0$ and $0 \leq \alpha < 1$.

Thus $\{x_n\}$ is a Cauchy sequence in X.

Since X being complete, let $x_n \to \xi$ for some $\xi \in X$.

Now for any integer k,

$$F_{x_n,T_k\xi}(\alpha p) = F_{T_1^m x_{n-1},T_k\xi}(\alpha p)$$

$$\geq F_{x_{n-1},T_k\xi}(p)$$

Taking limits on both sides, we get

$$F_{\xi,T_k\xi}(\alpha p) \geq F_{\xi,T_k\xi}(p)$$

If $\xi \neq T_k\xi$, a contradiction since $\alpha < 1$.

This implies that $\xi = T_k\xi$ for any integer k.

Thus ξ is a common fixed point of T_n, $n = 1,2,\ldots\ldots\ldots\ldots$

For the uniqueness of fixed point let if possible $\eta \neq \xi$ be another common fixed point of T_n, $n = 1,2,\ldots\ldots\ldots\ldots$

Then for each $p > 0$

$$F_{\xi,\eta}(\alpha p) = F_{T_1^m\xi,T_1^m\eta}(\alpha p)$$

$$\geq F_{\xi,\eta}(p)$$

which is impossible since $0 \leq \alpha < 1$.
Therefore $\xi = \eta$.
Hence ξ is an unique common fixed point of T_n, $n = 1,2,\ldots\ldots\ldots\ldots$

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Remark 1.4 : Vasuki[4] proved a fixed point theorem for a sequence of mappings for contractive iterates. In the above theorem we generalized the result of Vasuki[4] in Menger spaces.

Theorem 1.5 : Let $\{T_n\}$, $n = 1,2,\ldots\ldots\ldots\ldots$ be a sequence of self maps of complete Menger space (X, F, t) into itself with t (u, v) = min (u, v) for every u, v ∈ [0,1]. If for any two mappings T_1 and T_2, we have

$$(F_{T_1u,T_2v}(\alpha p))^2 \geq \min\{(F_{u,v}(p))^2, (F_{u,T_1u}(p))^2, (F_{v,T_1u}(p))^2, (F_{v,T_2v}(p))^2\}$$

for all u, v ∈ X, $p > 0$ and $0 \leq \alpha < 1$, then for $x_0 \in X$ the sequence $\{x_n\}$ is defined as

$$x_n = T_n x_{n-1}, \quad n = 1,2,\ldots\ldots\ldots\ldots$$

converges and its limit is the unique common fixed point of T_n.

Proof : Let x_0 be any arbitrary point in X. Then for each $p > 0$ and $0 \leq \alpha < 1$,

$$(F_{x_1,x_2}(\alpha p))^2 = (F_{T_1x_0,T_21}(\alpha p))^2$$

$$\geq \min\{(F_{x_0,x_1}(p))^2, (F_{x_0,T_1x_0}(p))^2, (F_{x_1,T_1x_0}(p))^2, (F_{x_1,T_2x_1}(p))^2\}$$

$$\geq \min\{(F_{x_0,x_1}(p))^2, (F_{x_0,x_1}(p))^2, (F_{x_1,x_1}(p))^2, (F_{x_1,x_2}(p))^2\}$$

$$\geq (F_{x_0,x_1}(p))^2$$

So $$(F_{x_1,x_2}(\alpha p))^2 \geq (F_{x_0,x_1}(p))^2$$

for each $p > 0$ and $0 \leq \alpha < 1$.

Suppose $F_{x_1,x_2}(\alpha p) \geq F_{x_0,x_1}(p)$

for each $p > 0$ and $0 \leq \alpha < 1$.

Again

$$(F_{x_2,x_3}(\alpha p))^2 = (F_{T_1x_1,T_2x_2}(\alpha p))^2$$

$$\geq \min\{(F_{x_1,x_2}(p))^2, (F_{x_1,T_1x_1}(p))^2, (F_{x_2,T_1x_1}(p))^2, (F_{x_2,T_2x_2}(p))^2\}$$

$$\geq \min\{(F_{x_1,x_2}(p))^2, (F_{x_1,x_2}(p))^2, (F_{x_2,x_2}(p))^2, (F_{x_2,x_3}(p))^2\}$$

$$\geq (F_{x_1,x_2}(p))^2$$

So $(F_{x_2,x_3}(\alpha p))^2 \geq (F_{x_1,x_2}(p))^2$
for each $p > 0$ and $0 \leq \alpha < 1$.
Again suppose $F_{x_2,x_3}(\alpha p) \geq F_{x_1,x_2}(p)$
for each $p > 0$ and $0 \leq \alpha < 1$.
Therefore by induction

$$F_{x_n,x_{n+1}}(\alpha p) \geq F_{x_{n-1},x_n}(p)$$

Thus $\{x_n\}$ is a Cauchy sequence in X.
Since X being complete, let $x_n \to \xi$ for some $\xi \in X$.
Now for any integer k,

$$(F_{x_n,T_k\xi}(\alpha p))^2 = (F_{T_n x_{n-1},T_k\xi}(\alpha p))^2$$

$$\geq \min\{(F_{x_{n-1},\xi}(p))^2, (F_{x_{n-1},T_n x_{n-1}}(p))^2, (F_{\xi,T_n x_{n-1}}(p))^2, (F_{\xi,T_k\xi}(p))^2\}$$

$$\geq \min\{(F_{x_{n-1},\xi}(p))^2, (F_{x_{n-1},x_n}(p))^2, (F_{\xi,x_n}(p))^2, (F_{\xi,T_k\xi}(p))^2\}$$

Taking limits on both sides, we get

$$(F_{x_n,T_k\xi}(\alpha p))^2 \geq \min\{(F_{\xi,\xi}(p))^2, (F_{\xi,\xi}(p))^2, (F_{\xi,\xi}(p))^2, (F_{\xi,T_k\xi}(p))^2\}$$

$$\geq (F_{\xi,T_k\xi}(p))^2$$

Suppose $F_{\xi,T_k\xi}(\alpha p) \geq F_{\xi,T_k\xi}(p)$

for each $p > 0$ and $0 \leq \alpha < 1$.

If $\xi \neq T_k\xi$, a contradiction since $\alpha < 1$.

CONCLUSION

This implies that $\xi = T_k\xi$ for any integer k.
Thus ξ is a common fixed point of T_n, $n = 1,2,\ldots\ldots\ldots\ldots$
For the uniqueness[8] of fixed point let $\eta \neq \xi$ be another common fixed point of
T_n, $n = 1,2,\ldots\ldots\ldots\ldots\ldots$
Then for each $p > 0$

$$(F_{\xi,\eta}(\alpha p))^2 = (F_{T_1\xi,T_2\eta}(\alpha p))^2$$

$$\geq \min\{(F_{\xi,\eta}(p))^2, (F_{\xi,T_1\xi}(p))^2, (F_{\eta,T_1\xi}(p))^2, (F_{\eta,T_2\eta}(p))^2\}$$

$$\geq \min\{(F_{\xi,\eta}(p))^2, (F_{\xi,\xi}(p))^2,$$

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$$(F_{\eta,\xi}\ (p))^2,\ (F_{\eta,\eta}\ (p))^2\ \}$$

$$\geq (F_{\xi,\eta}\ (p))^2$$

This is impossible.
Therefore $\xi = \eta$.
Hence ξ is an unique common fixed point of T_n , $n = 1,2,\ldots\ldots\ldots\ldots$

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