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Enhanced plasma acceleration by lighter species in a multi-component plasma sheath

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In many modern applications involving charge species acceleration in the plasma sheath region, substantial gain in efficiency is achievable by using complex mixtures of ion species, often accompanied by an empirical optimization. In the present study we obtain and characterize admissible plasma entry flow into an electrostatic sheath using a sheath width dependent dispersive version of the entry criterion in a multiple ion species plasma. The characterization is thus possible in terms of relative concentration of lighter and heavier ion species as well as sheath width, both of which are important controlling factors for the ion acceleration achievable by a stable sheath structure. The realistic cases where sheath structure must be confined to a finite dimension about the boundary are recovered to have substantially enhanced entry flow velocity values. The optimization usually achieved by variation in relative concentration of ion species, while keeping sheath width nearly constant, is thus estimated to result in higher acceleration and increased efficiency of the associated processes.

I. INTRODUCTION

The plasma sheath is an electrostatic structure ensuring outflow of an equal flux of ions and electrons although that the ions are much massive than the highly mobile electrons Clearly,thesheath structure at the boundary must accelerate ions in order to equalize theirflux with highly mobile electrons. In many plasma applications this ionacceleration by sheath is an ideal means for extracting high energy ion flux as well asmodifying the target surfaces such that higher ion flow velocities are usually desirable [1]. Single and multiple ion species plasmas are employed for diverse range of applications involving ion acceleration, for example from surface processing[2–5] to precision etching [6,7]and propulsion thrust generation for deep space exploration[8,9] to electrostatic confinement of complex fluids [10–14] and generation of negative ions for fusion plasma applications [15]. In a variety of laboratory conditions additional control and effciency of sheath acceleration are achievable by various empirical approaches and a greater clarity of the mechanism is desired in order to make a systematic optimization. In many applications it is observed that efficiency of surface modifications by a certain ion species is greatly improved on using a multiple species plasma, or by maintaining an optimum sheath width For example, the effciency of surface nitriding of steel is observed to improve many folds by using hydrogen [3]. Similarly the larger sheath widths are seen to result in smaller etching rates using an Argon ion etching process [16]. These examples show that the ion acceleration produced in plasma sheath in these complex conditions is strongly modified by upstream plasma constitution and the role

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of the selfconsistent variation of the plasma entry velocity needs to be analyzed by revisiting the sheath mode [17, 18].

In this paper we analyze the entry velocities in a two species plasma as function of concentration and sheath width. The sheath widths in many applications are comparable to and determined by the achievable plasma dimensions. Same is also applicable to various surface modification experiments where theplasmaconditionsare optimized by maintaining approximately similar sheath widths, largely dictated by plasma dimensions required to appropriately cover then on uniform target surfaces by the working plasma. In the present analysisa self consistent relationship between plasmaentry-velocity, ionspecies concentration and sheath width is obtained and characterized. The analysis present edheretreats the entry velocity of a homogeneous plasma with multiple ion species flowing with a common speed in to the electrostatic sheath consistent with usual observations [19,20]. Our analysis finds that increase in the lighter ion concentration potentially enhances the over all plasma entry velocity for comparable sheath-widths.

In the present paper, in Sec. II we obtain a lowest order relationship between sheath-width L_S and entry velocity for a multiple ion-species plasma which has a finite ratio of plasma Debye length to sheath width, λ_D/L_S. In Sec. III the dependence of entry plasma velocity into a two species nitrogen-hydrogen plasma sheath is characterized with respect to lighter ion concentration and a range of sheath width values. The analysis is extended to a plasmas with a range of two ion species combinations, including N-H, Ar-H, Xe-H and Ar-Xe. The results are summarized and conclusions are presented in Sec. IV

II. HYDRODYNAMIC MODEL OF FINITE WIDTH MULTISPECIS SHEATH

A sheath structure is generally viewed as a several Deby length wide space charge layer separating boundary from the bulk plasma whose inflow velocity V_{Entry} into the boundary must exceed the Bohm velocity [21], or the ion acoustic speed. Within the linear framework, that underlies the Bohm and generalized Bohm criteria, this discrete picture is greatly simplified and the Bohm velocity is recovered as the velocity with which the plasma must flow in order that the inward bound ion acoustic mode below a limitingk_s suffer cut off. This allows perturbation produced by an absorbing boundary to decay monotonically over a characteristic length $L_S{\sim}2\pi k_s^{-1}$ into the plasma, such that a bulk plasma can exist unperturbed by the boundary. In the linear terms, therefore, the region of width L_S localized at the boundary is suitably identifiable as sheath and no sudden parameter variation, or a sharp sheath-edge is invoked. Moreover, L_S is the characteristics length of potential drop and the non-neutral region is further localized at dimensions smaller than L_S such that there is no immediate breakdown of the quasineutrality at the sheath edge $x = L_S$ with respect to the boundary. The nonlinear effects are expected to bring gradual modifications allowing the width L_S and V_{Entry} to depend on the structure amplitude, or the sheath drop $\emptyset_s$. The linear prescription of L_S therefore remains useful lowest order approximation of its more advanced $\emptyset_s$ dependent forms [1] for analyzing the impact of many other lowest order factors, like the relative concentration of additional ion species.

The largely followed sheath width L_S estimation procedure involves using expression for space-

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charge limited current with potential drop $\emptyset_s$ and substituting in this expression the lowest order prescription for V_{Entry} namely, the Bohm velocity given by the standard sheath model [1]. Since we employ a generalized Bohm velocity which is a lowest order amplitude-independent value but suitably admit variation in multiple species concentration, we use the corresponding lowest order expression also for L_S for the self-consistency rather than using an alternate, space-charge-limited-current based, expression for finding the width L_S
In order to formulate this generalized entry velocity expression with sheath-width dependence ,we begin by considering an unmagnetized system of two ion species plasma steadily flowing into an absorbing material boundary with velocities $v_1$andv_2, respectively. Considering the finite ion momentum balance and negligible electron inertia limit corresponding to low phase velocity perturbations, $\omega/kv_{the} \ll 1$,where $v_{the} = \sqrt{T_e/m_e}$, we write the one dimensional electrostatic particle and momentum flux balance, respectively, for charged plasma species, α,

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha v_{x\alpha}) = 0 \qquad (1)$$

$$m_\alpha \frac{\partial V_\alpha}{\partial t} + m_\alpha V_\alpha . \nabla V_\alpha = q_s E - \nabla p_\alpha / n_\alpha \qquad (2)$$

And the equation of state providing closure for the fluid description,

$$\nabla p_\alpha = \gamma_\alpha T_\alpha \nabla n_\alpha \qquad (3)$$

where the subscript α represents electrons and multi ple ion species. The electric field **E** in (2) is given by the Poisson's equation,

$$\frac{\partial E}{\partial x} = 4\pi e\left(\sum_j n_s - n_e\right) \qquad (4)$$

where the subscript j represents various ion species. Up on linearization about an equilibrium defined as,

$$V_{s0},\ V_{e0} \parallel B,\ V_{s0} = V_0, \qquad \sum_j n_{j0} = n_{e0} = n_0,$$
$$E_0 = 0,$$

The linearized version of the model (2)-(4) is obtained for perturbed quantities written with subscript1,

$$\frac{\partial n_{j1}}{\partial t} + n_0 v_{jx1} = n_{j1} v_{0x} \qquad (5)$$

$$m_j \frac{\partial u_{jx1}}{\partial t} + m_j\left(v_{0x}\frac{\partial v_{jx1}}{\partial x}\right) = eE_{1x} - \gamma_j \frac{T_j}{n_0}\frac{\partial n_{j1}}{\partial x} \qquad (6)$$

$$\frac{\partial^2 \emptyset_1}{\partial x^2} = e\left(\sum_j n_{j1} - n_e\right) \qquad (7)$$

For a wave like spatial perturbation the above linearized equations yield the electrostatic dispersion relation governing the ion perturbationas [20].

$$k^2 + \frac{1}{\lambda_D^2} - \sum_j \frac{\omega_{pj}^2}{\left(\frac{\omega}{k} - \upsilon_j\right)^2 - c_j^2} = 0, \qquad (8)$$

Where ω_{pj}, υ_j and c_j are the plasma frequency, flow velocity and thermal velocity of the j'th ion species, respectively,and λ_D is the plasma Debye length.

Transforming the dispersion relation to the frame drifting with the wave velocity ω/k the dispersion relation (8) applies to a steadily flowing plasma with a stationary wave $(\omega/k = 0)$. Exploring the condition where no oscillatory perturbations from

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aboundary extend into the bulk requires $k^2 < 0$ in (8), yielding the generalized Bohm criterion [18],

$$\sum_j \frac{q_j^2 n_j}{m_j v_j^2 - \gamma_j T_j} \leq \frac{e^2 n_e}{T_e} \tag{9}$$

The condition (9) ensures that if a non-neutral perturbation is localized at a downstream location $x = 0$ the resulting potential variation is consistent with a net ion flux that balances the loss rate of electrons and quasineutrality at upstream locations is preserved. Note that the magnitude $|k^2|$ present in (8) but missing from the in equality (9) is the associated with the inversed squared sheath width which tends to be infinite for the marginal case of vanishing Debye lengths $\left(\lambda_D^2 \gg k^2\right)$ corresponding to equality form of the criterion (9).

The sheath structure supported by (9) is stationary in the frame of absorbing boundary and the plasma species must have flow velocities satisfying (9). The criterion (9) additionally admits ion species with distinct (non-equal) flow velocities below their instability threshold. However, the ion populations with a relative drift velocities experience an instability generated friction [19] and in most of the cases of practical interest tend to move with a common velocity [19, 22, 23]. The present analysis is therefore limited to examining only the stable cases of a common entry velocity for both the species and explores solutions with this effective entry velocity of the plasma into the sheath. For a two-ion species plasma with $j = 1, 2$ substituting $v_1 = v_2 = v_{Entry}$, in the cold plasma limit $\left(T_j \to 0\right)$ of the expression (8), applying the sheath conditions $\left({}^{\omega}/_{k} = 0\right)$ and $k^2 < 0$ and retaining $|k^2\lambda_D^2| \equiv |\lambda_D^2 L_S^2|$, yields,

$$V_{Entry} = \left[\frac{\sum_j \lambda_D^2 \omega_{pj}^2}{1 - \lambda_D^2 / L_S^2}\right]^{1/2} \tag{10}$$

Certain obvious conclusion can readily be drawn from the form of V_{Entry} given by (10) about the sheath structure.

These include that, (i) the real entry velocity V_{Entry} values are possible only in thel imit $L_s > \lambda_D$ meaning that sheath width shorter than the Debye length are not possible and λ_D is the smallest limiting value for the sheath dimensions for which V_{Entry} tends to infinity. (ii) The entry velocity equivalent to the Bohm velocity (for $j = 1$) or system ion acoustic speed $(j > 1)$ correspond to the other limiting case of vanishing λ_D, the entry velocity V_{Entry} needs to exceed the Bohm velocity for all realistic cases with $\lambda_D / L_s \neq 0$ and the criterion must be satisfied in its inequality form.

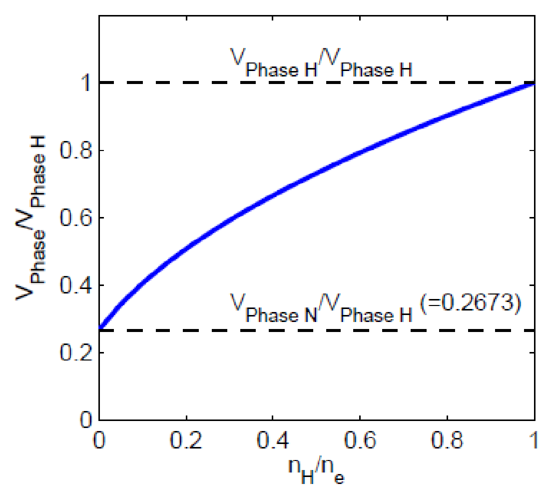

FIG.1:Reference variation of the effectve ion acoustic phase velocity (normalized to that in pure H plasma) in a plasma with increasing fraction of the Hconcentration replaced by the N concentration ${}^{n_H}/_{n_e}$

The marginal case $\lambda_D / L_s \to 0$ corresponds to sheath forming with infinite dimension as compared to the plasma Debye length.The value V_{Entry} in this case is equivalent to the effective ion acoustic velocity in a stationary multiple ion species plasma and serves as a good reference for the entry velocities required to produce a finite dimension sheath in a multiple ion

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species plasma. Considering its maximum relevance to nitriding experiments [3,6,7] we first examine for a two species plasma with $N^+$and H^+ ions using nitrogen (atomic mass ~ 14) and hydrogen (atomic mass ~ 1) for our characterization. The effective ion acoustic phase velocity (≡ entry flow velocity for $v_1 = v_2$) normalized to that in a pure hydrogen plasma is plotted as function of changing species concentration in Fig.1. The solid line in Fig.1 shows simplistic variation of the effective ion acoustic phase velocity in the bulk plasma where ion species are stationary.Clearly, with increasing concentration of H^+ ions, the phase velocity of ion acoustic mode in a pure nitrogen ion plasma (V_{PhaseN}/V_{PhaseH}~ 0.15 , indicated by bottom dashed line) shifts towards that of the pure hydrogen plasma (indicated by top dashed line) and approaches this value at 100 % concentration of H^+ ions. The phase velocity variation presented in Fig. 1 shows that with a hydrogen ion concentration ratio of half the $V_{Phase} \equiv V_{Entry}$ value required in a pure nitrogen plasma grows by about a factor of $2.74\left[\sim V_{Phase\left(n_H=\frac{1}{2}n_e\right)}/V_{Phase(n_H=0)} = 0.7316/0.2673\right]$ For the processing effciency depending on the incident velocity of the nitrogen ion the addition of lighter species like hydrogen can therefore enhance the effciency by a considerable factor, as often witnessed in many processing experiments.

The result in Fig.1 are however limited to assumptions that (i) the sheath dimensions are infinite as compared to the Debye length and (ii) the ratio λ_D/L_s is not sensitive enough to plasma condition such that sheath dimensions are not required to be adjusted as a result of changing ion species concentration. Since these assumptions may not be exactly applicable to most of the experimental cases, one may need to examine the cases that are more relevant to typical surface processing experiments. We consider cases where λ_D/L_s has a small but non zero (<1) value and the optimization is made by regaining a workable plasma sheath configuration each time the plasma conditions are changed. In many case, for example ,the optimization might involve changing plasma conditions while maintaining a glow region of approximately same dimension such that the plasma sheath of approximately same strength keeps covering the target surface.This methodology effectively involves readjustment of sheath width to anapproximately common value in newer plasma conditions, which must result in strong change of entry velocity according to (10). Clearly, the marginal case presented in Fig.1 addresses sensitivity of the entry velocity only to changing plasma,assuming that in each case the $\lambda_D/L_s \rightarrow 0$ is achievable. In many cases of large species mass ratio, however ,L_s may approach λ_D to a close factor for achieving the required stable conditions and very high V_{Entry} might result. The characterization of $V_{Entry}$with finite λ_D/L_s value is further done for such case sand presented in the following Sections.

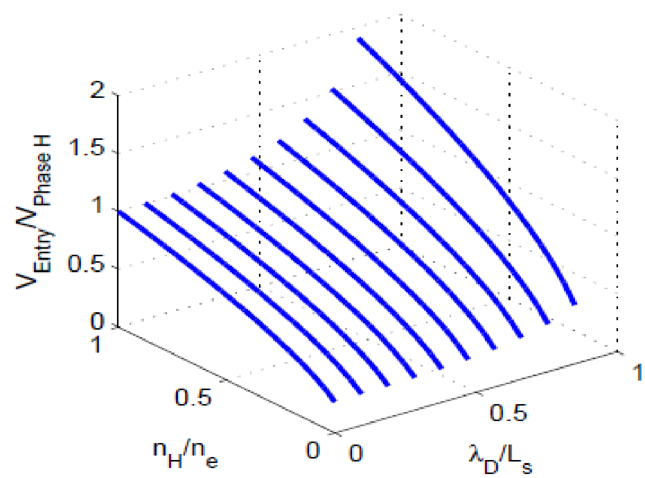

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FIG.2:Variation of common entry velocity of the ion species into the sheath with respect to ratio λ_D/L_s and normalized concentration $n_H/n_e$of H^+at the sheath edge

III. ENTRY FLOW VARIATION FOR FINITE SHEATH DIMENSIONS

In Fig.2 we have plotted the variation in the effective entry velocity of the species $v_{Entry} = v_1 = v_2$ into the sheath structure at the plasma sheath edge as a function of both ,the relative concentration of n_H(normalized to total electron density n_e) and the ratio of Debye length λ_D to sheath width L_s.The concentrationof H^+ is varied from 0 to100 % of the total ion density for the analysis. The selection of the range of this ratio is motivated by the condition that the ratio λ_D/L_s is bounded between 0 and1for all the valid cases of sheath formed in a plasma as the Debye length λ_D represents the minimum limit of the sheath width in a plasma. The profile at $\lambda_D/L_s \rightarrow 0$ corresponds to the variation of the effective entry velocity with respect to the increasing relative concentration of H^+ when sheath dimensions are very large compared to λ_D (almost infiniteλ_D) and entry velocity equal to the system ion acoustic velocity for respective n_H/n_e is sufficient to form a stable sheath. In sheath-physics terms,the limiting value k_s for this case is nearly zero meaning that sheath potential can penetrate to arbitrary lengths into the bulk plasma,although disallowing any oscillatory potential variation.The profiles with larger λ_D/L_s show increase in required value of entry velocity. This nature of variation implies that confining the sheath dimensions further to only few λ_D requires larger and larger entry velocities since they enable access to larger and larger limiting values,or allow formation of sheath with smaller and more practical L_s.With increasing ratio λ_D/L_s the V_{Entry}variation is more pronounced and steeper, indicating that even larger entry velocities are prescribed for the species when H^+concentration is large and sheath thickness begins to be finite and acquire more realistic values in terms of Debye length, rather than being infinity. Note that the usual quasineutral bulk plasma region will not be accessible in the case of limit $\lambda_D/L_s \rightarrow 0$. In most of the applications, therefore, the plasma must be operated away from this limit to have a limited sheath dimension and finite bulk plasma which provides a steady source of energetic ions. Maintaining these conditions by accessing the similar sheath dimensions with increased H^+ concentration, for example, will essentially allow the over all plasma, and hence N^+ions, to accelerate upto entry velocities many times higher than in the pure N^+ plasma at equivalent sheath

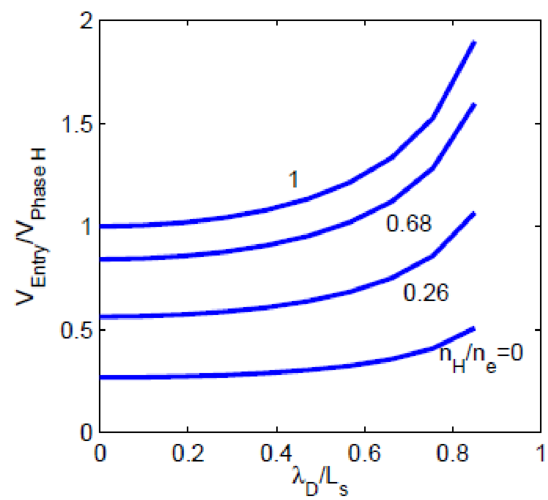

FIG.3:Entry velocity as a function of ratio λ_D/L_s in a two species plasma (N and H) for selected values of relative H concentration, n_H/n_e=0,0.26,0.68 and 1, respectively. The case n_H/n_e=0 corresponds to pure N plasma where as n_H/n_e=1 corresponds to a pure H plasma.

In order to more quantitatively examine the variation in the entry velocity with respect to the sheath thickness, the profiles of V_{Entry} at certain selected values of H^+concentration are plotted in Fig.3. This can be noted that for sheath length approaching infinity ($\lambda_D/L_s \rightarrow 0$), the entry velocity reduces to phase velocity of the ion acoustic mode for the corresponding plasma presented in Fig.1. However a sharper gain is achieved in entry velocity with reducing sheath thickness than with increasing the concentration of lighter H ion species.

The entry velocities are considerably larger for the sheath thickness approaching its limiting value λ_D. The present analysis is limited to covering effects only of collisionless ion species and to the cases where the collisions with the neutrals and molecular species are less effective, especially in the region of sheath,or in the region $x < L_s$ where the absorbing boundary is located at $x = 0$. Given that we have used a generalized formulating multiple ion species the effect of presence of additional ion species can be accommodated rather conveniently in the present analysis Considering that nitrogen plasmas in surface processing experiments also feature a large percentage of N^+ ions [3], we additionally examined the effect of an equal fraction of N^+ species.

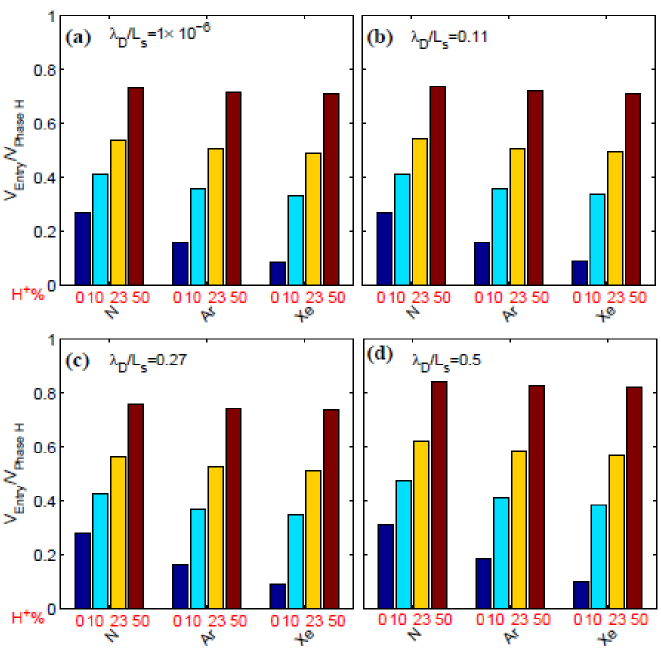

FIG.4: Entry velocity enhancement as a function of ratio λ_D/L_s and concentration of H^+ ions for various ion combinations species plasma (a)N and H, (b)Ar and H,and (c)Xe and H, for the values of relative H concentration, n_H/n_e = 0,0.1,0.23 and 0.5, respectively

The presence of N^+ lowers the over all plasma entry velocity marginally (not presented here). However, no significant impact on the large acceleration generated by the presence of hydrogen was observed for relevant cases when H^+ are present in considerable fraction ($\sim$ 50%) of the total ion population.

We further present relative acceleration generated by presence of lighter hydrogen ions in various heavier ion species plasmas other than nitrogen. InFig.4 the entry velocities of two species plasmas formed with combinations, N-H, Ar-H and Xe-H are presented as function of increasing values of concentration of H^+ ions. Same normalization for V_{Entry} in all the combinations is used for an easy comparison.Three vertical set of four bars in each subplots correspond to plasma with species N, Ar and Xe respectively, while the four bars in each set correspond to percentage H^+ion concentration values 0,10, 23, and 50%, respectively.Subplots (a), (b), (c) and (d) correspond to values of λ_D/L_s=1×10^{-6}, 0.11, 0.27 and 0.5, respectively.This can be seen that for each value of $\lambda_D/L_s$the acceleration of plasma is enhanced instable sheath condition that must be achieved during the plasma operations when percentage of lighter ion concentration, namely, $H^+$is increased.The maximum acceleration in heavier ion entry velocity is achieved in the cases where the sheath dimensions are maintained at the smallest value or where $L_s$approaches values comparable to λ_D.An acceleration of N^+species appears possible from usual valueof$\sim 0.28V_{PhaseH}$ to an increased value$>$ $0.8V_{PhaseH}$, for example, when $\lambda_D/L_s = 0.5$ is maintained.The relative acceleration is maximum for heaviest species ,i.e., for Xe for which the entry velocity enhances from $\sim 0.1V_{PhaseH}$ to $>$ $0.8V_{PhaseH}$ for $\lambda_D/L_s = 0.5$ and 50% H^+

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concentration such that again of about a factor of 8 is achieved. Note that even for the plasmas with no presence of H^+(first bar in each set) the acceleration is marginally enhanced with larger λ_D/L_s, or with smaller sheath widths.

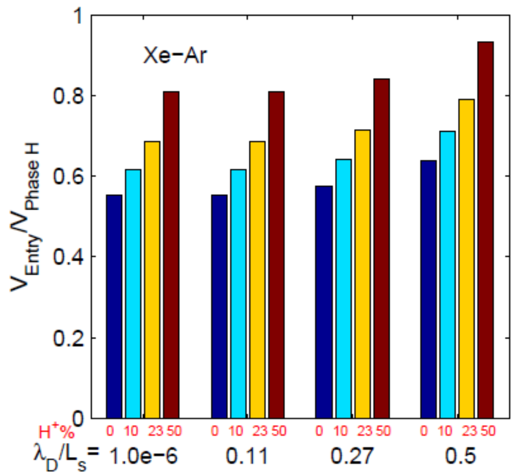

FIG.5:Entry velocity enhancement as a function of ratio λ_D/L_s and concentration of Ar^+ ions for the ion combination Ar-Xe. Bars in each set correspond to the values of relativeAr concentrations, n_{Ar}/n_e =0,0.1,0.23and0.5,respectively.

Analysis of an additional, rather analytic combinationis finally included where Ar ions are is present in a Xe plasma. This is a case relevant to usual sheath physics experiments, where a rather simpler setup, free from the chemical and atomic-molecular processes is desired and explored. The entry velocity in Ar-Xe plasma with varying concentration of Ar+ ions is presented in Fig. 5. Four sets of vertical bars corresponding to λ_D/L_s, = 1 × 10−6, 0.11, 0.27 and 0.5, are presented where four bars in each set represent the entry velocity corresponding to the values of Ar^+ ion concentration, 0, 10, 23 and 50 %, respectively. It is evident that because of smaller mass ratio of Ar to H, the acceleration remains relatively moderate in this case and the V_{Entry} values, for example, with 50% Ar^+ (> 0.820.28VPhase Hat finite λ_D/L_s remain in agreement with the experimental values. As already mentioned, a larger impact of chemical and atomic-molecular processes is expected in more complex processing plasmas that involve more chemically reactive species, e.g., in the plasma with ion combination N-H [3].

IV. SUMMARY AND CONCLUSIONS

To summarize, we presented application of linear dispersive property of a multiple species plasma to estimate required presheath acceleration of the ion species. The entry velocity of the ions in various two species plasma, involving N^+, Ar^+ and Xe^+in combination with H^+ions, is characterized to illustrate the dependence of the common entry velocity of these ion species on (i) the relative concentration of the species and (ii) the sheath thickness expressed in terms of plasma Debye length. Results of the analysis are presented to show that when friction between ion species is suffciently strong enabling them to flow into a sheath with a common entry velocity, the presence of a lighter H^+ ion in a N plasma results in the increase of the entry velocity of the plasma to the electrostatic sheath. In a variety of laboratory conditions where multiple ion species plasmas may be employed for diverse range of applications, for example from surface processing to precision etching and thrust generation for space propulsion, the ion accelerating plasma sheath region may be confined to a region of finite width in terms of the plasma Debye length. For such spatially limited of sheath formations dispersive properties of plasma is used to show that the plasma flow velocity into the sheath region are governed by a version of generalized sheath criterion accounting for the effects of finite sheath thickness. The effect of constant sheath dimension usually maintained in many experimental conditions might result in a sharper increase in the entry velocity as the bulk plasma conditions are modified for the optimization of efficiency of application. The characterization

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presented provides quantitative estimation of behavior of plasma sheath with respect to optimization parameters like plasma composition and sheath structure width.The increasing concentration of lighter species is a considerable factor in resulting higher accelerations of the plasma flux into the sheath. A sharper increase is however noted resulting from the reducing sheath thickness. For a rather application relevant N-H ion species combination, an enhancement by a factor ~ 2.7 is estimated in the common entry velocity of the plasma with 50% concentration of lighter, H^+ ion species. A larger enhancement is recovered in for the combination of larger mass ratio. In an Xe-H plasma. for example, this factor is estimated to grow up to 8 for sheath dimensions when the sheath thickness approaches a close multiple (~ 2) of the Debye length. The weak effect of reducing sheath thickness is recoverable also in pure, single species, limit of the combinations when lighter ion (H^+) concentration approaches zero. The present study limited to use of linear dispersion exclude modifications arising from the effects of finite amplitude and kinetic properties of the particle relation, distributions. Finite effects of magnetic field are also need to be explored for sheath structures forming normal to the magnetic field. Inclusion of these effects suggest potential future extension of the present analysis along with the application of present results to multiple species plasma experiments in the real laboratory set ups.For the special cases where various ion species flow into the sheath structure with a common velocity owing to a large friction, it is shown that variation in their concentrations and masses can produce significant variations in the scales of the resulting sheath structure. The sheath structure in a self-consistent set up where the plasma is produced by the shock due to the moving boundary is excluded from the present analysis and should be addressed in the future analysis.

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