

## **Manpower Levels for Business with Various Recruitment Rates in Twelve point State space through Stochastic Models**

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### **Abstract**

In the human resource management Manpower had been a significant role for the business. Currently, human resources especially administrative tasks have been affected by the selection for an industry business model. The HR manager will be given their best to achieve their goals. The aim of this paper is, to find the steady state of crisis and steady state of probabilities for the manpower with different conditions with full and nil level of business. The basic assumptions and their model have been given based on the transition probabilities with different parameters.

**Keywords:** Manpower, Markov chain, Steady state, Crisis rate

### **1. Introduction**

Manpower planning is very essential for the human resources for the recruitment in an organization. Any organization generally runs on profit-making needs based on the organization responsibility for the period of business with manpower. Recruitment can complete when the movement is full. In some situations organization may face crisis based on the full manpower or nil manpower for the business. Many problems in manpower have been discussed by Bartholomew [1-2]. To find the promotion and wastages with size was given by Leeson, G.W (1982) [3]. Applications of Manpower levels in Business with different states; six and eight point state space has been discussed by R.Arumugam and M.Rajathi [4-5]. V.Subramaniam [6] focused the optimal policy for promotion, training and wastages. An introduction of training time and the confidence intervals with limits for the stationary rate given by Yadhavalli et. al [7]. Approach to manpower problems are dealt in lots of opportunity methods as early as 1947, the take a look at of probabilistic or random approach of manpower problem is very

substantial crucial. Many probabilistic methods of manpower structures are studied and projected typically with the beyond Vajda et al. [8]. An application of stochastic models for manpower levels for business has been discussed by Dr.R.Arumugam [9]. Yadavalli and Natarajan [10] deliberated a semi-markov model; stochastic model and applications. The random approach of the manpower problem was focused with the impact of pressure on promotion by Yadavalli et al. [11]. R.Arumugam et. al [12] focused on applications of manpower with various recruitment rates in the stochastic model. Markov's model is implemented in an examining the nature of manpower systems in the variety of personnel in each grade or age profile of workers under a variety of situations and comparing rules for dominant manpower structures (for example, Young and Almond [13], Young [14], Bartholomew [15] and Gani [16]). Finally, prediction of Markov chain model has been discussed by Dr.R.Arumugam et. al [17].

## **2. Different States with Markov Model**

Manpower, Business and recruitment are three characteristics focused in this paper. The given three characteristics may be fully available, moderately available and zero availability; it may lead to the business with full or nil level. Here, we are representing the characteristics particularly business and the steady state state of the crisis and probability of the crisis state are also studied with various levels. Numerical examples are also given.

## **3. Assumptions of this Model**

Some important assumptions are given below for this model,

- i. We have two levels of manpower viz., full and nil.
- ii. Two levels of business, namely full and nil;

## **4. Analysis**

The continuous time in markov chain in stochastic process  $X(t)$  determines the system with eleven point state space as given below in the order of manpower, business and recruitment.

The sample space of the model is given below.

$$S = \{(0\ 0\ 0), (0\ 0\ 1), (0\ 1\ 0), (0\ 1\ 1), (0\ 0\ 2), (0\ 1\ 2), (1\ 0\ 0), (1\ 0\ 1), (1\ 0\ 2), (1\ 1\ 0), (1\ 1\ 1), (1\ 1\ 2)\} \longrightarrow (1)$$

$$Q = \begin{bmatrix} M/B/R & (0\ 0\ 0) & (0\ 0\ 1) & (0\ 1\ 0) & (0\ 1\ 1) & (0\ 0\ 2) & (0\ 1\ 2) & (1\ 0\ 1) & (1\ 0\ 1) & (1\ 0\ 2) & (1\ 1\ 0) & (1\ 1\ 1) & (1\ 1\ 2) \\ (0\ 0\ 0) & \gamma_1 & \beta_{001} & \beta_{010} & \mu_{001} & 0 & 0 & 0 & 0 & 0 & \beta_{110} & 0 & 0 \\ (0\ 0\ 1) & \alpha_{001} & \gamma_2 & \beta_{011} & 0 & \mu_{012} & 0 & \beta_{100} & 0 & a & 0 & 0 & 0 \\ (0\ 1\ 0) & \alpha_{010} & \alpha_{011} & \gamma_3 & 0 & \beta_{012} & 0 & \mu_{100} & 0 & 0 & 0 & 0 & 0 \\ (0\ 1\ 1) & \lambda_{011} & 0 & 0 & \gamma_4 & \beta_{012} & 0 & \beta_{111} & 0 & 0 & \beta_{120} & 0 & 0 \\ (0\ 0\ 2) & 0 & \lambda_{002} & 0 & \alpha_{012} & \gamma_5 & 0 & 0 & 0 & 0 & \beta_{112} & 0 & 0 \\ (1\ 1\ 2) & 0 & 0 & \lambda_{110} & 0 & \alpha_{102} & \gamma_6 & 0 & \beta_{101} & 0 & 0 & 0 & 0 \\ (1\ 0\ 0) & 0 & \lambda_{001} & 0 & \lambda_{110} & a & 0 & \gamma_7 & 0 & \beta_{102} & 0 & 0 & 0 \\ (1\ 0\ 1) & \alpha_{102} & 0 & \lambda_{112} & 0 & 0 & 0 & \alpha_{112} & \gamma_8 & 0 & 0 & 0 & 0 \\ (1\ 0\ 2) & \alpha_{110} & 0 & 0 & \alpha_{111} & \alpha_{112} & a & 0 & 0 & \gamma_9 & 0 & 0 & 0 \\ (1\ 1\ 0) & \alpha_{112} & 0 & \alpha_{120} & 0 & 0 & 0 & a & b & 0 & \gamma_{10} & 0 & 0 \\ (1\ 1\ 1) & a & \alpha_{111} & 0 & 0 & 0 & \alpha_{102} & 0 & 0 & 0 & \lambda_{112} & \gamma_{11} & \\ (1\ 1\ 2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{12} \end{bmatrix} \quad \text{---(2)}$$

In this model the 1<sup>st</sup> co-ordinate represents the availability/ non-availability of manpower. 2<sup>nd</sup> co-ordinate represents the business and the 3<sup>rd</sup> co-ordinate represents recruitment level.

Where,

$$\gamma_1 = -(\beta_{001} + \beta_{010} + \mu_{011} + \beta_{110}) \quad \text{---(3)}, \quad \gamma_2 = -(\alpha_{001} + \beta_{011} + \mu_{012} + \beta_{100}) \quad \text{---(4)}$$

$$\gamma_3 = -(\alpha_{010} + \alpha_{011} + \mu_{100} + \beta_{012}) \quad \text{---(5)}, \quad \gamma_4 = -(\lambda_{011} + \beta_{012} + \beta_{111} + \beta_{120}) \quad \text{---(6)}$$

$$\gamma_5 = -(\lambda_{002} + \alpha_{012} + b + \beta_{112}) \quad \text{---(7)}, \quad \gamma_6 = -(\lambda_{110} + b + \alpha_{102} + \beta_{101}) \quad \text{---(8)}$$

$$\gamma_7 = -(\lambda_{001} + \lambda_{110} + a + \beta_{102}) \quad \text{---(9)}, \quad \gamma_8 = -(\alpha_{102} + \lambda_{112} + \alpha_{112} + a) \quad \text{---(10)}$$

$$\gamma_9 = -(\alpha_{110} + a + \alpha_{111} + \alpha_{112}) \quad \text{---(11)}, \quad \gamma_{10} = -(\alpha_{112} + a + b + \alpha_{120}) \quad \text{---(12)}$$

$$\gamma_{11} = -(\alpha_{102} + a + \alpha_{111} + \lambda_{112}) \quad \text{---(13)}, \quad \gamma_{12} = -(\alpha_{102} + a + \lambda_{112}) \quad \text{---(14)}$$

$$\text{Let } \pi = [\pi_{000} \pi_{001} \pi_{010} \pi_{011} \pi_{002} \pi_{012} \pi_{100} \pi_{101} \pi_{102} \pi_{110} \pi_{111} \pi_{112}]$$

$$\pi Q = 0, \quad \pi e = 1 \quad \text{---(15)}$$

From equation (15), we obtain the steady state probabilities

$$\pi_{000} = \frac{d_0 \lambda_{100}}{z \sum d_i} \quad \text{---(16)}, \quad \pi_{001} = \frac{d_1 \lambda_{101}}{z \sum d_i} \quad \text{---(17)}$$

$$\pi_{010} = \frac{d_2 \lambda_{102}}{z \sum d_i} \quad \text{---(18)}, \quad \pi_{011} = \frac{d_0 \mu_{000}}{z \sum d_i} \quad \text{---(19)}$$

$$\pi_{002} = \frac{d_1 \mu_{101}}{z \sum d_i} \quad \text{---(20)}, \quad \pi_{100} = \frac{d_2 \mu_{102}}{z \sum d_i} \quad \text{---(21)}$$

$$\pi_{101} = \frac{d_0 \beta_{112}}{z \sum d_i} \quad \text{---(22)}, \quad \pi_{102} = \frac{d_1 \beta_{112}}{z \sum d_i} \quad \text{---(23)}$$

$$\pi_{110} = \frac{d_2 \beta_{112}}{Z \sum d_i} \quad \text{-----} (24), \quad \pi_{111} = \frac{d_0 \alpha_{111}}{Z \sum d_i} \quad \text{-----} (25)$$

$$\pi_{012} = \frac{d_1 \alpha_{012}}{Z \sum d_i} \quad \text{-----} (26)$$

$$\pi_{112} = \frac{d_1 \alpha_{111}}{Z \sum d_i} \quad \text{-----} (27)$$

Where,  $d_0 = \beta_{100} \alpha_{112} + \beta_{111} \alpha_{112} + \alpha_{111} \alpha_{102}$ ;  $d_1 = \alpha_{100} \beta_{102} + \alpha_{101} \alpha_{112} + \alpha_{102} \alpha_{111}$ ;

$d_2 = \alpha_{101} \alpha_{102} + \alpha_{102} \beta_{101} + \alpha_{102} \alpha_{111}$ .  $\sum_{i=0}^2 d_i = [d_0 + d_1 + d_2]$  and  $Z = [a + b]$

The crisis occurs in the following states,  
 {(0 1 1), (0 1 2), (1 1 0), (1 1 1), (1 1 2)}

$$C_\infty = \beta_{011} \pi_{110} + \beta_{012} \pi_{111} + \lambda_{110} \pi_{112} + \lambda_{111} \pi_{011} + \lambda_{112} \pi_{012} \quad \text{-----} (28)$$

Based on the steady state probability, we can obtain

$$C_\infty = \frac{\lambda_{101}}{Z \left[ \sum_{i=0}^2 d_i \right]} \left[ \mu_{101} (d_0 \beta_{102} + d_1 \beta_{112} + d_2 \lambda_{110}) + d_0 \beta_{101} \lambda_{001} + d_1 \alpha_{111} \lambda_{100} \right] \quad \text{-----} (29)$$

## 5. Numerical Calculation for the Steady State Cost

### Twelve Point State Space

#### Case (i)

The steady state probabilities and the rate of crisis are estimated by utilizing the equation (28) and (29) individually.

Taking,  $a = 12$ ,  $b = 5$ ,  $\alpha = 4$ ,  $\beta = 5$ ,  $\beta_{100} = 13$ ,  $\alpha_{102} = 2$ ,  $\alpha_{112} = 8$ ,  $\alpha_{111} = 8$ ,

$\alpha_{101} = 7$ ,  $\alpha_{100} = 9$ ,  $\beta_{101} = 2$ ,  $\beta_{102} = 15$ ,  $\lambda_{100} = 7$ ,  $\lambda_{101} = 12$ ,  $\lambda_{102} = 10$ ,  $\mu_{100} = 12$ ,  $\mu_{101} = 7$ ,  
 $\mu_{102} = 5$ ,  $\beta_{011} = 4$ ,  $\lambda_{110} = 3$ ,  $\lambda_{111} = 7$  and  $\beta_{112} = 10$ .

we get  $\pi_{000} = 0.1824224$ ,  $\pi_{001} = 0.463272$ ,  $\pi_{010} = 0.149842$ ,  $\pi_{011} = 0.347163$ ,  $\pi_{002} = 0.287628$ ,  
 $\pi_{102} = 0.397331$ ,  $\pi_{100} = 0.029821$ ,  $\pi_{101} = 0.315742$ ,  $\pi_{102} = 0.29474$ ,  $\pi_{110} = 0.180642$ ,  
 $\pi_{111} = 0.17703$ , and  $\pi_{112} = 0.193316$ .

#### Case (ii)

If we assume that a and b values,

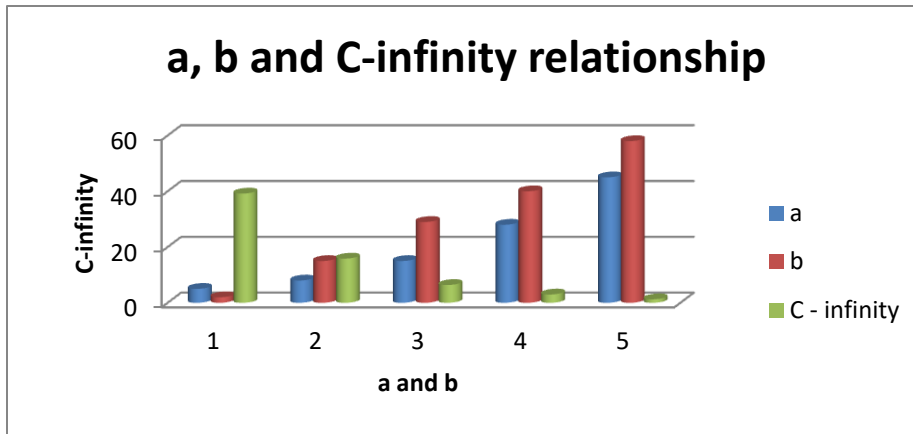
$\beta = 3$ ,  $\beta_{100} = 15$ ,  $\alpha_{102} = 10$ ,  $\alpha_{112} = 12$ ,  $\alpha_{111} = 5$ ,

$\alpha_{101} = 12$ ,  $\alpha_{100} = 15$ ,  $\beta_{101} = 5$ ,  $\beta_{102} = 15$ ,  $\lambda_{100} = 12$ ,  $\lambda_{101} = 15$ ,  $\lambda_{102} = 14$ ,  $\mu_{100} = 17$ ,  $\mu_{101} = 9$ ,

$\mu_{102} = 7$ ,  $\beta_{011} = 3$ ,  $\lambda_{110} = 6$ ,  $\lambda_{111} = 12$  and  $\beta_{112} = 10$ .

**Table: 1 Relationship among a, b and  $C_\infty$**

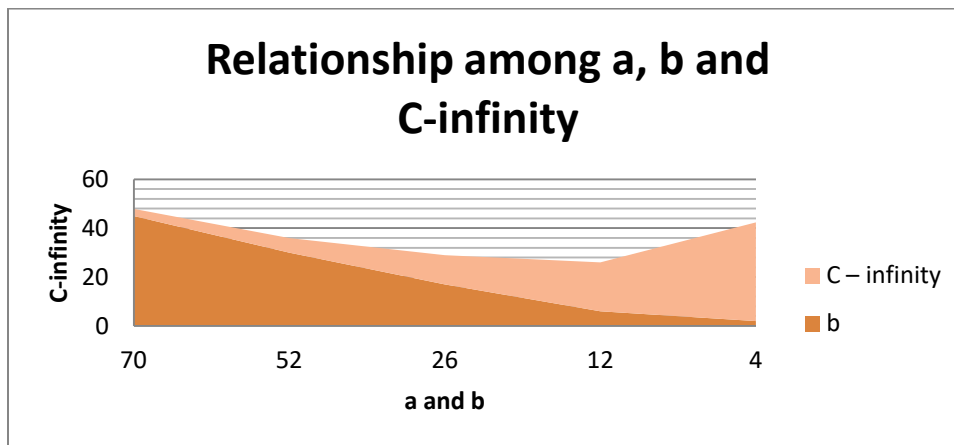
a	5	8	15	28	45
b	2	15	29	40	58
C - infinity	39.1860	15.8673	6.3915	2.9825	1.2586



**Figure 1: a, b and  $C_{\infty}$  Relationship**

**Table 2 Relationship among a, b and  $C_{\infty}$**

a	70	52	26	12	4
b	45	30	17	6	2
C – infinity	2.9264	5.9775	11.9643	20.0145	40.3927



**Figure 2: a, b and  $C_{\infty}$  Relationship**

**Steady State Cost**

The Steady state cost calculated from the following formula

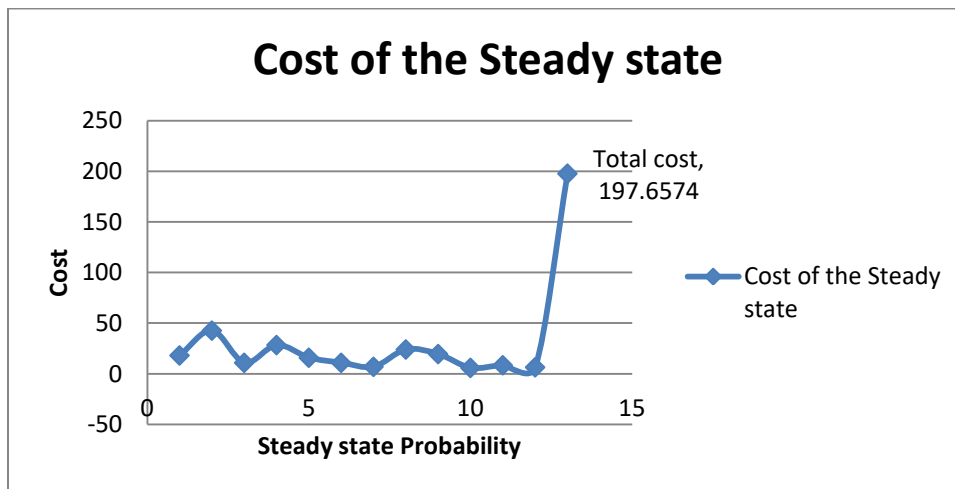
$$C_{ijk} = \pi_{ijk} (c_{MP}^i + c_B^j + c_R^k)$$

Here,  $c_{MP}^i$  means manpower cost at the state  $i = 0$  or  $i = 1$ ,  $c_B^j$  means business cost at the states  $j = 0$  or  $j = 1$  and  $c_R^k$  means the recruitment or enrollment or departure at the states  $k = 1$  or  $k = 2$ . We assume the various costs,

$$c_{MP}^0 = 32, c_{MP}^1 = 12, c_B^0 = 25, c_B^1 = 12, c_R^0 = 33 \text{ and } c_R^1 = 45$$

**Table 3 Relationship between Steady state probability and cost of the steady state**

Steady state probability	Cost of the Steady state
$\pi_{000} = 0.153231$	18.0931
$\pi_{001} = 0.42841$	42.6379
$\pi_{010} = 0.15912$	10.8642
$\pi_{011} = 0.33751$	28.3667
$\pi_{002} = 0.22973$	15.8651
$\pi_{102} = 0.19157$	10.8643
$\pi_{100} = 0.05731$	6.8793
$\pi_{101} = 0.28135$	23.9852
$\pi_{102} = 0.30754$	19.4608
$\pi_{110} = 0.15042$	5.8252
$\pi_{111} = 0.13729$	8.4219
$\pi_{112} = 0.16727$	6.3937
<b>Total cost</b>	<b>197.6572</b>



**Figure 3: Cost of the Steady state**

## **6. Conclusion**

The above first figure and first table represents that the relationship among a, b and C-infinity, When both a and b are increasing and the corresponding C-infinity decreasing. Figure two and table two shows that if, both a and b are decreasing and therefore the crisis rate increasing. Therefore, the instance of the steady state cost represents there's no business amid the amount.

When the business starts, there's problem occurs within the steady state cost. From the figure three, the steady state cost illustrates that there's moderate business amid the amount. Exactly when the business starts, there's a fluctuations which occurs within the steady state cost. From this, we found that the cost of the steady state increasing, when there's full business additionally recruitment rate getting increased. At the purpose when there's no business, the steady state cost gets increasing and therefore the corresponding recruitment rate gets increasing. However, it's seen that if there's full business and recruitment rate gets increasing but the steady state cost gets decreasing.

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